

Stat 471: Lecture 14

Monte-Carlo Inference II.

Moo K. Chung
mchung@stat.wisc.edu

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1. Estimating type I error. When doing inference, your decision can be wrong in two ways. The first is a *type I error*, which occurs when we reject H_0 when in fact it is true. $P(\text{type I error}) = P(\text{reject } H_0 | H_0 \text{ true}) = \alpha$. The resulting α is called the significance level of the test. In the `strength.data` example, we reject H_0 if $t < -2.37$ or $t > 0.91$ at $\alpha = 0.1$ level.

```
>> quantile(t,[0.1 0.9])
ans = -1.9975  0.5598
```

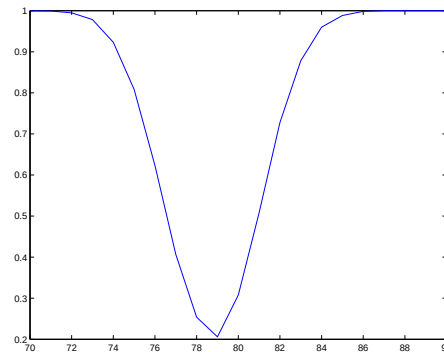
So the 0.2 level test would be to reject H_0 if $t < -2.00$ or $t > 0.56$.

2. The P -value is the smallest level of significance at which H_0 would be rejected (You are supposed to know this fact- STAT312). The smaller the P -value, it is easier to reject H_0 . Since our observed $t = -0.72$, we did not reject H_0 . In this example, let us find the P -value. We reject H_0 if $t < c_L$ or $t > c_R$ at α level. If $c_L = -0.73$, we can not reject H_0 . So $c_L \geq -0.72$ and if c_L becomes large, the significance level increases. So the minimum significance level is obtained when $c_L = -0.72$.

```
>> size(find(sort(t)<=-0.72),1)/m
ans = 0.4963
>> 2*(0.5-0.0037)
ans = 0.0074
```

3. Estimating Type II error. A *type II error* occurs when we fail to reject H_0 when H_0 is actually false.

$P(\text{type II error}) = P(\text{not reject } H_0 | H_0 \text{ false}) = \beta$. be on bootstrap.



The statistical power of a test is defined as

$$\text{Power} = 1 - \beta.$$

It is the probability of not making a type II error and it measures the ability of the test to detect a false null hypothesis. It can be estimated by counting the number of times the test fail to reject H_0 when H_0 is false. Let's compute the power for `strength.data` example with $\alpha = 0.2$. We assume $\mu = \mu_0 \neq 80$ and use the statistic $T = (\bar{X} - \mu)/(S/\sqrt{n})$. The rejection rule if T is not in $(-1.9975, 0.5598)$.

```
n=size(arm,1); m=10000;
cL=-1.9975; cU=0.5598;
for i=70:90
    mu_a=i;
    for j=1:m
        x=sigma*snrnd(n)+mu_a;
        t(j)=(mean(x)-80)/(std(x)/sqrt(n));
    end;
    beta(i)= sum((cL<t)&(t<cU))/m;
end;
plot([70:90],1-beta(70:90))
```

4. Read Chapter 6 Bootstrap. Lecture 15-20 will be on bootstrap.