

Stat 471: Lecture 22

MCMC II.

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1. *Random-walk Metropolis.* The Metropolis sampler refers to the original version of Metropolis *et al.* (1953). For proposal distribution, we assume $q(y|x) = q(|y - x|)$. See example $Y \sim N(x, 1)$ in lecture 20. In this case,

$$\alpha(X_i, Y) = \min\left(\frac{\pi(Y)}{\pi(X_i)}, 1\right).$$

Given $X_i = x_i$,

1. Generate $Y \sim g(|y - x_i|)$.
2. $X_{i+1} = Y$ with probability $\alpha(X_i, Y)$ and $X_{i+1} = X_i$ otherwise.

Generating $Y \sim g(y - x_i)$ is equivalent to $Y = X_i + \epsilon_i$, where $\epsilon_i \sim g(y)$. So we have random walk $X_{i+1} = X_i + B_i \epsilon_i$ where B_i is Bernoulli with $P(B = 1) = \alpha$.

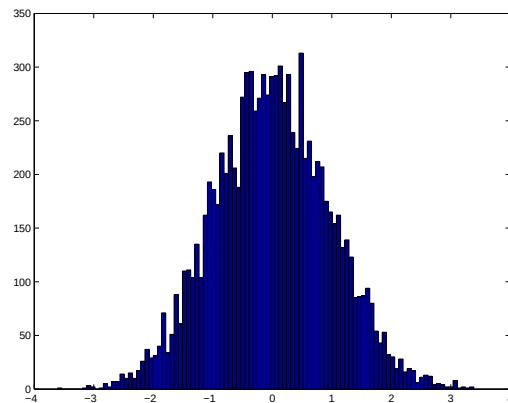
Let's generate $N(0, 1)$ from $Unif(-2, 2)$.

```
n=10000;  
x(1)=0;  
snrpdf=inline('exp(-x^2/2)');  
for i=2:n  
    y=x(i-1) + 2*(1-2*rand);  
    alpha=min([1, snrpdf(y)  
               /snrpdf(x(i-1))]);  
    u=rand;  
    if u <= alpha  
        x(i)=y;  
    else  
        x(i)=x(i-1);  
    end  
end
```

2. *Independent sampler* is the special case of H-M algorithm (Tierney, 1994). Proposal distribution q is independent of x , i.e. $q(y|x) = q(y)$. In this case

$$\alpha(X_i, Y) = \min\left(\frac{\pi(Y)}{\pi(X_i)} \frac{q(X_i)}{q(Y)}, 1\right).$$

This method does may not work well if q is very similar to π . Given X_i ,



1. Generate $Y \sim q(y)$.
2. $X_{i+1} = Y$ with probability $\alpha(X_i, Y)$ and $X_{i+1} = X_i$ otherwise.

The independent sampler can be compared to Accept-reject method for generating random numbers. In AR method,

1. Generate $Y \sim q(y)$
2. $X = Y$ with probability $\frac{\pi(Y)}{cq(Y)}$.

It can be shown that the independent Metropolis-Hastings algorithm is more efficient than the Accept-Reject method since independent M-H will accept more proposed values(see Robert and Casella, Monte Carlo Statistical Method for detail).

3. The *Gibbs sampler* is a special case of the Metropolis-Hastings algorithm but it differ in two ways:
 1. We always accept Y (candidate point).
 2. Need to know full conditional distributions.

By construction, the Gibbs sampler is multidimensional. Here we will only consider 2-dimensional case. Given joint density $f(x, y)$, the Gibbs sampler is a way to generate the marginal density $f_X(x) = \int f(x, y) dy$ and $f_Y(y) = \int f(x, y) dx$.

We will study the Gibbs sampler in lecture 23.