

Stat 471: Lecture 28

Bivariate smoothing.

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1. We studied how to simulate scalar, vector data. To simulate continuous stochastic processes (lecture 29), we need a concept of smoothing. Let us study kernel smoothing in $\mathbb{R}^k$. Let $y_1, \dots, y_m$ be the neighboring data in close proximity to data y_0 . Then Nadaraya-Watson version (1964) of kernel smoothing is defined as mapping

$$y_0 \rightarrow \sum_{i=0}^m w_i y_i$$

where $\sum_{i=0}^m w_i = 1$. We need to choose w_i such that w_i measures the contribution of data y_i . The contribution should decrease as the distance between y_i and y_0 increases. So our first guess in designing kernel would be to use

$$w_i(y_i, y_0) = \exp\left(-\frac{\|y_i - y_0\|}{2\sigma^2}\right).$$

It guarantee the condition that the contribution of data y_i decreases but it will not make the total sum of weights equal to 1. So we normalized it such that

$$w_i(y_i, y_0) = \frac{\exp\left(-\frac{\|y_i - y_0\|}{2\sigma^2}\right)}{\sum_{i=0}^m \exp\left(-\frac{\|y_i - y_0\|}{2\sigma^2}\right)}.$$

σ^2 controls the amount of smoothing.

2. Suppose we have spatial data y_i collected at point p_i . We assume the following bivariate model

$$Y_i = g(p_i, \sigma^2) + \epsilon_i$$

where we assume ϵ_i are iid. We are interested in estimating g . We estimate g by performing the bivariate kernel smoothing.

```
[px,py] = meshgrid([-3:0.1:3]);
g = px.*exp(-px.^2-py.^2);
mesh(g);
e=normrnd(0,0.2,61,61);
y=g+e;
kernel=inline('exp(-(x.^2+y.^2)/2)')
dx=kron(ones(3,1),[-1:1])
dy=kron(ones(1,3),[-1:1])
>>weight=kernel(dx,dy)/sum(sum(kernel(dx,dy)))
    0.0751    0.1238    0.0751
    0.1238    0.2042    0.1238
    0.0751    0.1238    0.0751
temp_y=y;
for i=1:10
    smooth_y=conv2(temp_y, weight,'same');
    temp_y=smooth_y;
end;
```

