

Stat 992: Lecture 30

Curve Modelling III.

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1. *Active contour.* Active contour models (snakes) first proposed by Kass, Witkin and Terzopoulos (International Journal of Computer Vision 1:321-31, 1987) is a variational method for detecting object boundaries in images. Given n points $C^0 = \{p_1^0, \dots, p_n^0\}$ that define the initial closed contour, we deform the points to lie along the object boundary (Figure 1). The initial contour C^0 is deformed to $C^1 = \{p_1^1, \dots, p_n^1\}$ by minimizing a certain energy function. We deform contour $C_i = \{p_1^i, \dots, p_n^i\}$ to $C_{i+1} = \{p_1^{i+1}, \dots, p_n^{i+1}\}$ by

$$p_j^{i+1} = p_j^i + d(p_j^i) \text{ for all } j$$

where d is the displacement vector that minimize the total energy. This procedure is iteratively performed until the snake C^i hit the object boundary.

2. *Energy via parameterization.* Read G. Sapiro's Geometric Partial Differential Equations and Image Analysis. Let $X(p)$ be a parameterization of contour C and I be a image intensity. Then the energy given by Kass *et al.* is

$$E(C) = \alpha \int |X'(p)|^2 + \beta \int |X''(p)| - \lambda \int |\nabla I(X(p))|.$$

The first two terms are the internal energy while the last term is the external energy. The internal energy is responsible for smoothness while the external energy is responsible for attracting the contour toward the object boundary. Parameters α, β and λ are determined a priori. Smaller λ reduces the noise but can not capture the sharp corners while larger λ can effectively capture boundary but it is sensitive to the noise. The the total energy given by Kass *et al.* can be written as

$$E(C) = E_{\text{internal}}(C) + \lambda E_{\text{external}}(C).$$

It can be shown from the *calculus of variations*, the contour should satisfy Euler-Lagrange equation:

$$-\frac{d}{dp}(\alpha X') + \frac{d^2}{dp^2}(\beta X'') - \lambda \Delta I(X) = 0.$$



Figure 1: Left: Initial contour C_0 picked manually using `ginput` mouse command in MATLAB. Middle: the trace of deformation. Right: After 50 iterations with no external energy term ($\lambda = 0$).

3. *Discrete version.* The energy corresponding to each points is denoted as

$$E(p_j^i) = E_{\text{internal}}(p_j^i) + E_{\text{external}}(p_j^i).$$

Energy terms are

$$E_{\text{internal}} = \sum_{j=1}^n E_{\text{internal}}(p_j^i),$$

$$E_{\text{external}} = \sum_{j=1}^n E_{\text{external}}(p_j^i).$$

The first internal energy term $\int |X'(p)|^2$ measures the linear elastic potential of a string that is proportional to the squared distance between neighboring points, i.e.

$$E_{\text{elastic}}(p_j^i) = \|p_{j+1}^i - p_j^i\|^2 + \|p_j^i - p_{j-1}^i\|^2.$$

The second term $\int |X''(p)|^2$ measures the curvature and at each point it is given as

$$E_{\text{curvature}}(p_j^i) = \|p_{j+1}^i - 2p_j^i + p_{j-1}^i\|^2.$$

Many authors incorporate different energy terms.

The external energy can be written

$$E_{\text{external}} = aE_{\text{intensity}} + bE_{\text{gradient}}$$

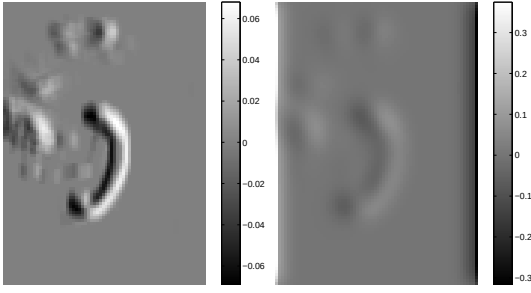


Figure 2: Left: $\partial_x I$, Right: $\partial_x(K_\sigma * I)$ Extreme image gradient corresponds to the object boundary.

where $E_{intensity}$ attracts the contour to either high or low intensity regions and $E_{gradient}$ forces the contour to move toward the extreme image gradient. $E_{intensity}(p_j^i) = f(I(p_j^i))$ for some simple function f . $E_{gradient}(p_j^i) = \|\nabla I(p_j^i)\|^2$. To make this energy function smooth, $\|\nabla K_\sigma * I(p_j^i)\|^2$ is used instead.

4. *Updating scheme.* Hamarneh and Gustavsson (Combining snakes and active shape models for segmenting the human left ventricle in echocardiographic images, IEEE computers in cardiology, 2000) uses the following updating scheme.

$$p_j^{i+1} = p_j^i + d(p_j^i) \text{ for all } j$$

where the displacement vector field is given in terms of force

$$d(p_j^i) = -\nu(F_{internal} + F_{external}).$$

This method would be very useful for improving upon manual segmentation result.

5. *Goodness of fit of contouring* The measure of the goodness of fit between the manual contouring and active contouring is based on the geodesic distance the snake has moved. Suppose that C_m gives the final estimate of the object boundary from the original manual contouring C_0 . Note that the path of point p_j^0 is $p_j^0 \rightarrow p_j^1 \rightarrow \dots \rightarrow p_j^m$ so the total geodesic distance the point moved is

$$d_{geodesic}(p_j^0, p_j^m) = \sum_{i=0}^{m-1} \|p_j^{i+1} - p_j^i\|.$$

For small deformation,

$$d_{geodesic}(p_j^0, p_j^m) \doteq \|p_j^m - p_j^0\|$$

and the total goodness-of-fit measure of the manual contouring is given by

$$GOF = \sum_{j=1}^n d_{geodesic}(p_j^0, p_j^m).$$

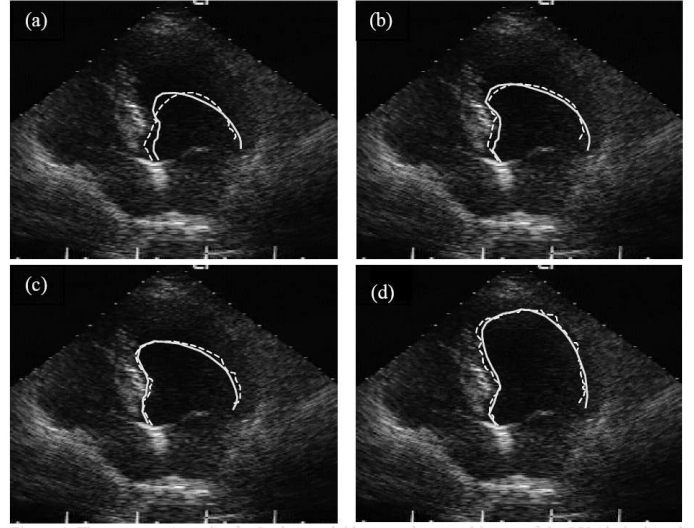


Figure 3: The result of Hamarneh and Gustavsson's method applied to an ultrasound image. See the original paper.

Problem 44. The following code will pick out the manual contouring points on images. To stop picking points, right click the mouse button.

```
imshow(image);
hold on;
xs = []; ys = [];
xold = 0; yold = 0;
but = 1;
while but == 1
    [xi, yi, but] = ginput(1);
    xs = [xs; xi];
    ys = [ys; yi];
    if xold;
        plot([xold xi], [yold yi], 'ro-');
    else
        plot(xi, yi, 'ro');
    end;
    xold = xi;
    yold = yi;
end;
plot([xi xs(1)], [yi ys(1)], 'r-');
```

Suppose your image is very simple binary image of a circle, i.e. $I(x, y) = 1$ if $x^2 + y^2 = r^2$ and $I(x, y) = 0$ if $x^2 + y^2 \neq r^2$. Pick out 10 points C_0 close to the circle boundary and write very simple active contouring MATLAB code for detecting a circle.