

Stat 992: Lecture 41

Image Analysis via Random Fields and Kernel Smoothing: Overview

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Although it may be difficult to formulate all the problems in image analysis in terms of random fields and kernel smoothing, it can be done non the less. I put references that I am familiar with so these are not the complete list of bibliography. Most references are cross-listed.

3. Gaussian mixture. This is widely used statistical model for image segmentation. Most methods combines EM algorithm with a Bayesian prior.

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4 and 5 Non mixture-based segmentation. Neural-network based segmentations are often used. Deformable surface tend to segment geometrically complex boundaries nicely.

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8. Image Registration. Most interesting image registration algorithms find applications in medical imaging.

1. Incorporating Prior Knowledge into Image Registration Ashburner J, Neelin P, Collins DL, Evans AC, Friston KJ (1997) *NeuroImage* 6:344-352.
2. Nonlinear Spatial Normalization using Basis Functions Ashburner J, Friston KJ (1999) *Human Brain Mapping* 7(4):254-266
3. Spatial Registration and Normalization of Images Friston KJ, Ashburner J, Poline JB, Frith CD, Heather JD, Frackowiak RSJ (1995) *Human Brain Mapping* 2:165-189.
4. Ashburner, J., Friston, K.J. 1997. Multimodal image coregistration and partitioning - a unified framework, *NeuroImage* 6:209-217.
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9. Random Fields. It is natural to model image as continuous functional data due to extremely high sampling density. Average brain image consists of $200 \times 200 \times 100 = 2$ million voxels or more.

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10. Linear additive model and other statistical models Most references are on fMRI time series analysis but should be applicable on other imaging modalities.

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8. Characterizing Evoked Hemodynamics with fMRI Friston KJ, Frith CD, Turner R, Frackowiak RSJ (1995) Neuroimage 2:157-165.
9. A Multivariate Analysis of PET Activation Studies Friston KJ, Poline J-B, Holmes AP, Frith CD, Frackowiak RSJ (1996) Human Brain Mapping 4:140-151.
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11. Curve modeling It could mean many things. Since a curve can be characterized by curvature, it includes the estimation of curvature and stochastic modeling on curvature function.

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14. Diffusion smoothing. There are a lot of diffusion smoothing paper in computer vision but none of them use it as a way to improve Gaussianness.

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16 Directional Smoothing and other smoothing techniques Smoothing vector and tensor fields

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17 False discovery rate (FDR) and 19 Permutation test This is a somewhat new method for performing multiple comparisons. It is not as popular as the random field theory in neuroimaging.

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