# Discussion 10

*Example* 1. A diary scientist conducted a study on the effect of two dietary supplements (formulated based on different mineral compositions) on anemia. Four cows were randomly selected to receive supplement A and four to supplement B. The data are composite mineral concentration (in $\mu g/d/Li$) measured on blood samples from each cow taken after three months using the supplement.

| | | | | |
|---|---|---|---|---|
| Supplement A | 5.7 | 7.1 | 6.6 | 5.0 |
| Supplement B | 5.8 | 4.7 | 3.8 | 5.3 |

(a) Find a 90% confidence interval for the difference between the mean concentration in the blood for cows on supplement A and the mean concentration for cows on B.
(b) Based *only* on the confidence interval calculated in part (a), what conclusion can you reach about a test of the null hypothesis that the mean concentration in the blood for cows on A is exactly 0.3 larger than the mean concentration for cows on B (versus the two-sided alternative). Explain.

*Example* 2. A study was conducted to compare two different exercise programs in terms of weight loss in adult men. Twelve men were available for this study. These men were divided into six groups of 2 men so that the two men in each group were similar in age and initial weight. One man in each group was assigned to each exercise program. The weight losses (in pounds) for all 12 men were recorded after six weeks.

| Group | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| Program A | 7 | 4 | 11 | 8 | 3 | 12 |
| Program B | 10 | 9 | 11 | 7 | 6 | 13 |

(a) Perform a test of the null hypothesis of equal efficacy for the two programs versus the two-sided alternative. Interpret the results.
(b) State the assumption underlying the test you use in part (a). *You do NOT need to justify these assumptions.*

*Example* 3. This question concerns the comparison of two different rejection rules for evaluating a hypothesis. The data consist of measurements of the breaking strength of wooden boards. For each board, an increasing force is applied until the board breaks. The measurement is the amount of force required to break the board.

A random sample of 100 observations of breaking force is available. Assume that these observations follow a $N(\mu, 800)$ distribution. Of interest is the null hypothesis $H_0 : \mu = 56$ versus the alternative $H_0 : \mu > 56$. Two different rejection rules are being considered; each has roughly the same value of $\alpha$.

**Rule A:** Reject $\mathrm{H}_0$ if $\bar{X} > 60$.

**Rule B:** For each observation, $X_i$, declare the observation to be "defective" if $X_i > 70$. Reject $H_0$ if the number of defective observations is $\geq 36$.

If, in fact, $\mu = 62$, find the power for each rule and determine which is more powerful.

*Example* 4. A scientist wishes to compare two drugs in terms of the glucose levels in the anterior chambers of the eyes of dogs. There will be $n$ dogs in the study. For each dog, one of the drugs will be randomly assigned to the right eye and the other to the left. The difference in glucose levels are known to be approximately normally distributed with a variance of 1.8 $(mg/dLi)^2$. The null hypothesis is that the two drugs have the same effect versus the one-sided alternative that drug B results in a higher glucose level. The hypothesis will be rejected if the observed difference between the sample mean for drug B and that for Drug A is 0.3 $mg/dLi$ or greater. How large does $n$ need to be so that there is a power of 90% when the true difference in glucose level is 0.5 $mg/dLi$?

## Solutions

*Solution* 1. This is a two-independent sample situation.
(a) Let $Y_A$ and $Y_B$ be the concentrations on A and B respectively and let $\mu_A$ and $\mu_B$ be the respective population means. The needed summary statistics are $\bar{y}_A = 6.1$, $s_A^2 = 0.8733$, $\bar{y}_B = 4.9$, and $S_B^2 = 0.74$. Then, due to the balanced data, $s_p^2 = (s_A^2 + s_B^2)/2 = 0.8067$. Since $s_p^2$ has 6 df, the appropriate t-value is 1.943. Hence, the CI is $(-0.03, 2.43)$.
(b) The null hypothesis is written: $H_0 : \mu_A = \mu_B = 0.3$. We notice that the value of 0/3 is contained within the CI above. Thus, we can conclude that the p-value $> 0.10$ for the given test.

*Solution* 2. This is a paired design, with the paring based on the age and initial weight. Let $D = Y_B - Y_A$, where $Y_A$ and $Y_B$ are the weight losses within a group for Program A and B respectively. We get $\bar{d} = 1.833$ and $s_d^2 = 4.966$. Hence, we have

$$t = \frac{1.833}{\sqrt{4.966/6}} = 2.015 \text{ on 5 df.}$$

Then, p-value$=2 \times P(T \geq 2.015) \approx 0.10$. There is very marginal evidence of difference between the two programs.

*Solution* 3. For Rule A: $\bar{X} \sim N(62, 800/100)$, and the power is

$$\begin{aligned}&P(\bar{X} > 60)\\ &= P\left(Z > \frac{60-62}{\sqrt{8}}\right)\\ &= P(Z > -0.71)\\ &= 0.7611.\end{aligned}$$

For Rule B, we have

$$\begin{aligned}&P(X_i > 70)\\ &= P\left(Z > \frac{70-62}{\sqrt{800}}\right)\\ &= P(Z > 0.28)\\ &= 0.3817.\end{aligned}$$

Denote the number of the defective observations as $W$. Then $W \sim B(100, 0.3817) \approx N(38.97, 23.78)$, and the power is

$$\begin{aligned}&P(W > 36)\\ &= P\left(Z > \frac{36-38.97}{\sqrt{23.78}}\right)\\ &= P(Z > -0.61)\\ &= 0.7291.\end{aligned}$$

Hence, Rule A is more powerful.

*Solution* 4. The random variable of interest in $D$, the difference in glucose levels between $B$ and $A$. The stated condition requires the distribution of $D$ under the given alternative. In this case, $D \sim N(.5, 1.8)$. The given condition is that $P(\bar{D} \geq 0.3|\mu_D = 0.5) = 0.9$. We know $P(Z \geq -1.28) = 0.9$. Thus, $-1.28 = (.3 - .5)/\sqrt{1.8/n}$. Solving results in $\sqrt{n} = 8.59$ and, rounding up, $n = 74$.