Assignment #2 — Due Friday, September 17, 2010, by 4:00 P.M.
Turn in homework to your TA's mailbox using this sheet as the cover page.

Fill in your name and also circle the *lecture section in which you are registered* and circle the *discussion section you expect to attend* to pick up this assignment.

Name:

Lecture 1 (Larget).	**311:** Tu 1:00 - 2:15pm	**312:** Th 8:00 - 9:15am	**313:** We 1:00 - 2:15pm
Lecture 2 (Hanlon).	**321:** Tu 1:00 - 2:15pm	**322:** We 2:30 - 3:45pm	**323:** We 1:00 - 2:15pm

Please answer the following questions.

1. For each part, determine if the random variable is binomial or not. If so, state the values of the parameters n and p. If not, explain which assumption(s) of the binomial distribution are violated.

   (a) There is an antibiotic which is known to be 70% effective in treating a common bacteria. In a clinical study, the antibiotic is administered to 120 unrelated patients with this bacteria. Let X_1 be the number of patients in the study successfully treated with the antibiotic.

   (b) Assume that in litters of mice that each mouse is equally likely to be male or female, independent of other mice in the litter, but the total number of mice in the litter is random and is equally likely to be any number from 8 to 14. Let X_2 be the number of female mice in a given litter.

   (c) Seeds from one supplier are known to germinate with probability 90% and seeds from a second supplier germinate with probability 93%. A researcher plants 50 seeds of each type. Let X_3 be the total number of seeds that germinate.

   (d) A research forest has 5000 trees, of which 250 are oaks. A researcher uses a random number generator to sample 400 different trees. Let X_4 be the number of oak trees in the sample.

   (e) In a genetic cross between fruit flies, male progeny can be one of four genotypes, AB, Ab, aB, and ab. A genetics model predicts these genotypes to be equally likely. There are 28 male flies produced in the cross. If this model is correct, let X_5 be the number of male flies with genotype AB.

2. In lecture, we gave general formulas for the mean and variance of a discrete random variable X. Namely,

$$\mathsf{E}(X) = \sum_k k\mathsf{P}(X = k), \tag{1}$$

and

$$\mathsf{Var}(X) = \mathsf{E}\Big(\big(X - \mathsf{E}(X)\big)^2\Big) = \mathsf{E}(X^2) - \big(\mathsf{E}(X)\big)^2 = \sum_k \big(k - \mathsf{E}(X)\big)^2 \mathsf{P}(X = k) \tag{2}$$

We also gave formulas for the mean and variance for a binomial random variable. Namely, if $X \sim \text{Binomial}(n, p)$, we have

$$\mathsf{E}(X) = np, \tag{3}$$

and

$$\mathsf{Var}(X) = np(1 - p). \tag{4}$$

The goal of this exercise is to show that the general formulas (1) and (2) give the same answers as the specific formulas (3) and (4), *when dealing with a binomial random variable.*

Specifically, consider $X \sim \text{Binomial}(5, 0.7)$.

(a) Compute $\mathsf{E}(X)$ using *both* formulas (1) and (3).
(b) Compute $\mathsf{Var}(X)$ using *both* formulas (2) and (4).

3. Consider a random variable X defined by the following distribution

k	-2	1	3	5
$\mathsf{P}(X = k)$	0.4	0.3	0.1	0.2

Compute $\mathsf{E}(X)$, $\mathsf{Var}(X)$, and $\mathsf{SD}(X)$.

4. (Continue the above example). We are often interested in questions such as *what is the probability that a random variable takes a value within one standard deviation of its mean?* Or similarly within two standard deviations of its mean. Consider the random variable X defined in the previous question. Compute the following.

(a) $\mathsf{P}\Big(\mathsf{E}(X) - \mathsf{SD}(X) \le X \le \mathsf{E}(X) + \mathsf{SD}(X)\Big)$
(b) $\mathsf{P}\Big(\mathsf{E}(X) - 2\mathsf{SD}(X) \le X \le \mathsf{E}(X) + 2\mathsf{SD}(X)\Big)$

5. p. 171, Problem 17 in the textbook.