

**Assignment #2 — Due Friday, September 23 by 4:00 P.M.**

Turn in homework to your TA's mailbox using this sheet as the cover page.

Fill in your name and also circle the *lecture section in which you are registered* and circle the *discussion section you expect to attend* to pick up this assignment.

**Name:**

**Lecture 1 (Hanlon).**      **311:** Tu 1:00 - 2:15pm      **312:** Th 8:00 - 9:15am      **313:** We 1:00 - 2:15pm

**Lecture 2 (Larget).**      **321:** Tu 1:00 - 2:15pm      **322:** We 2:30 - 3:45pm      **323:** We 1:00 - 2:15pm

Please answer the following questions.

- For each part, determine if the random variable is binomial or not. If so, state the values of the parameters  $n$  and  $p$ . If not, explain which assumption(s) of the binomial distribution are violated.
    - There is an antibiotic which is known to be 80% effective in treating a common bacteria. In a clinical study, the antibiotic is administered to 150 unrelated patients with this bacteria. Let  $X_1$  be the number of patients in the study successfully treated with the antibiotic.
    - Assume that in litters of mice that each mouse is equally likely to be male or female, independent of other mice in the litter, but the total number of mice in the litter is random and is equally likely to be any number from 6 to 12. Let  $X_2$  be the number of female mice in a given litter.
    - Seeds from one supplier are known to germinate with probability 93% and seeds from a second supplier germinate with probability 96%. A researcher plants 50 seeds of each type. Let  $X_3$  be the total number of seeds that germinate.
    - A research forest has 5000 trees, of which 250 are oaks. A researcher uses a random number generator to sample 400 different trees (a simple random sample, without replacement). Let  $X_4$  be the number of oak trees in the sample.
    - In a genetic cross between fruit flies, male progeny can be one of four genotypes,  $AB$ ,  $Ab$ ,  $aB$ , and  $ab$ . A genetics model predicts these genotypes to be equally likely. There are 28 male flies produced in the cross. If this model is correct, let  $X_5$  be the number of male flies with genotype  $AB$ .
  - In the United States, 42% of the population has type A blood. Consider taking a sample of size 4. Let  $X$  denote the number of persons in the sample with type A blood. Compute the following probabilities.
    - $P(X = 0)$
    - $P(X = 2)$
    - $P(0 \leq X \leq 2)$
    - $P(0 < X \leq 2)$
  - In the United States, 30% of individuals have “superior” distance vision, in the sense of scoring 20/15 or better on a standardized vision test without class. We collect a random sample of size  $n$  from this population. Let  $\hat{p}$  represent the sample proportion of individuals with superior vision.
    - Let  $n = 2$ . Give the sampling distribution of  $\hat{p}$ .
    - Let  $n = 20$ . Compute  $E(\hat{p})$ ,  $\text{Var}(\hat{p})$ ,  $\text{SE}(\hat{p})$ .
-

(c) Let  $n = 20$ . Compute  $P(\hat{p} = .25)$ .

4. Consider a random variable  $X$  defined by the following distribution

$k$	-4	-1	3	5	11
$P(X = k)$	0.1	0.3	0.1	0.2	.3

Compute the following.

- (a)  $E(X)$ ,  $\text{Var}(X)$ , and  $\text{SD}(X)$ .
- (b)  $P(E(X) - \text{SD}(X) \leq X \leq E(X) + \text{SD}(X))$ .
- (c)  $P(E(X) - 2\text{SD}(X) \leq X \leq E(X) + 2\text{SD}(X))$ .

5. Consider three random variables  $X_1, X_2, X_3$  where  $E(X_1) = 2$ ,  $E(X_2) = 11$ , and  $E(X_3) = -3$ . Also,  $E(X_1^2) = 20$ ,  $E(X_2^2) = 140$ ,  $E(X_3^2) = 31$ . Calculate the following.

- (a)  $E(3X_1 + 4X_2 - 2X_3)$
- (b)  $E(X_1 + 2X_1^2 - X_2 + X_2^2 - 3X_3^2)$
- (c)  $\text{Var}(X_1)$ ,  $\text{Var}(X_2)$ , and  $\text{Var}(X_3)$ .
- (d) Additionally assume that  $X_1, X_2$ , and  $X_3$  are independent. Compute

$$\text{Var}(3X_1 + 4X_2 - 2X_3).$$

(e) Consider a fourth random variable  $X_4$  with  $E(X_4) = 4$ . Is it possible that  $E(X_4^2) = 12$ ? Explain your answer.

6. In lecture, we gave general formulas for the mean and variance of a discrete random variable  $X$ . Namely,

$$E(X) = \sum_k kP(X = k), \tag{1}$$

and

$$\text{Var}(X) = E\left((X - E(X))^2\right) = E(X^2) - \left(E(X)\right)^2. \tag{2}$$

We also gave formulas for the mean and variance for a binomial random variable. Namely, if  $X \sim \text{Binomial}(n, p)$ , we have

$$E(X) = np, \tag{3}$$

and

$$\text{Var}(X) = np(1 - p). \tag{4}$$

The goal of this exercise is to show that the general formulas (1) and (2) give the same answers as the specific formulas (3) and (4), *when dealing with a binomial random variable*.

Specifically, consider  $X \sim \text{Binomial}(3, 0.6)$ .

- (a) Compute  $E(X)$  using *both* formulas (1) and (3).
- (b) Compute  $\text{Var}(X)$  using *both* formulas (2) and (4).

7. p. 171, Problem 17 in the textbook.