Assignment #3 — Due Friday, September 30 by 4:00 P.M.
Turn in homework to your TA's mailbox using this sheet as the cover page.

Fill in your name and also circle the *lecture section in which you are registered* and circle the *discussion section you expect to attend* to pick up this assignment.

Name:

Lecture 1 (Hanlon).   **311:** Tu 1:00 - 2:15pm   **312:** Th 8:00 - 9:15am   **313:** We 1:00 - 2:15pm

Lecture 2 (Larget).   **321:** Tu 1:00 - 2:15pm   **322:** We 2:30 - 3:45pm   **323:** We 1:00 - 2:15pm

Please answer the following questions.

1. The following questions ask you to compute a point estimate and confidence interval for an unknown proportion p. We assume that the data comes from a binomial model $X \sim Binomial(n, p)$. We let n denote the number of trials and x the number of observed successes in the data.

   (a) $n = 10$, $x = 3$. Compute a point estimate and 95% confidence interval for p.
   (b) $n = 50$, $x = 24$. Compute a point estimate and 95% confidence interval for p.
   (c) $n = 1000$, $x = 635$. Compute a point estimate and 95% confidence interval for p.

2. The following questions ask you to compute a p-value for a binomial test. We assume that the data comes from a binomial model $X \sim Binomial(n, p)$. We let n denote the number of trials and x the number of observed successes in the data.
   Note: To compute a p-value for a two-sided test use the method used in the Example on Slide 75 (in the proportion notes).

   (a) $n = 10$, $x = 4$. Compute the p-value for the test of $H_0 : p = .5$ vs. $H_A : p \neq .5$.
   (b) $n = 50$, $x = 21$. Compute the p-value for the test of $H_0 : p = .35$ vs. $H_A : p > .35$.
   (c) $n = 37$, $x = 16$. Compute the p-value for the test of $H_0 : p = .45$ vs. $H_A : p < .45$.

3. For this course you need to be able to compute binomial probabilities by hand and with a computer (for example using R). In this problem compute the following probabilities with a computer. In your solutions note which software you used and give the command used to generate your answers. In a particular genetic cross, each offspring has white flowers with probability $p = 0.4$ and yellow flowers with probability $1 - p = 0.6$, and colors are determined independently for each individual offspring. There are 1000 offspring. Find:

   (a) The probability that exactly 380 offspring have white flowers.
   (b) The probability that the number of offspring with white flowers is between 370 and 390, inclusive.
   (c) The probability that 380 or fewer offspring have white flowers.
   (d) The probability that 380 or fewer offspring have white flowers if, in fact, $p = 0.37$ instead of 0.4.

   We suggest that you use R for solving this problem and provide a summary of the relevant commands here. R has several built in functions for binomial random variable calculations. Specifically, the function `dbinom` computes probabilities of single outcomes and the function `pbinom` computes the cumulative distribution function of the binomial distribution. Each function takes three arguments - the outcome, n, and p. As an

example of the use of R, suppose that a fair die is rolled 1000 times and X is the number of #6's while Y is the number of even die rolls. The following R commands compute these probabilities.

Probability	R commands
$P(X = 150)$	`dbinom(150,1000,1/6)`
$P(X \leq 150)$	`pbinom(150,1000,1/6)` or `sum(dbinom(0:150,1000,1/6))`
$P(X < 150)$	`pbinom(149,1000,1/6)` or `sum(dbinom(0:149,1000,1/6))`
$P(Y > 510)$	`1 - pbinom(510,1000,1/2)` or `sum(dbinom(511:1000,1000,1/2))`
$P(Y \geq 510)$	`1 - pbinom(509,1000,1/2)` or `sum(dbinom(510:1000,1000,1/2))`
$P(480 < Y \leq 510)$	`pbinom(510,1000,1/2) - pbinom(480,1000,1/2)` or `sum(dbinom(481:510,1000,1/2))`

4. Consider a discrete random variable X with $\mathsf{E}(X) = -.3$. The probability distribution of X is given by the following table.

k	-10	?	3	5	7
$\mathsf{P}(X = k)$	0.2	0.3	0.2	?	0.1

Fill in the missing values in the table.

5. BRCA1 is a gene that has been linked to breast cancer. Researchers used DNA analysis to search for BRCA1 mutations in 169 women with family histories of breast cancer. Of the 169 women tested, 27 had BRCA1 mutations. Let p denote the probability that a woman with a family history of breast cancer will have a BRCA1 mutation. Compute a point estimate and a 95% confidence interval for p. Interpret the confidence interval in the context of the study.

6. The brown recluse spider *Loxosceles reclusa* is common in North America and has a nasty bite. In a study of the spider's diet preferences, researchers gave each of 141 spiders a choice between a live cricket and a dead cricket. They found that 98 spiders chose the dead cricket and 43 chose the live one. Use the binomial test to test the null hypothesis that these spiders have no preference between live and dead crickets versus the alternative that they prefer dead crickets. State hypotheses, find a test statistic, and compute a p-value. Summarize your findings in the context of the problem.

7. p. 170, Problem 14 in the textbook.

8. p. 170, Problem 15 in the textbook.