Regarding Item 2. in hw1:

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2. by lecture 5  Riesz and Sz.Nagy (RN),
   pgs 242-246, statement of the
   Mercer-Hilbert-Schmidt Theorem, in
   .../pf2/riesz.nagy.pdf.
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The Mercer-Hilbert-Schmidt Theorem is on pg 243 of the file in pdf2/riesz.nagy

Recall that the eigenvalue eigenvector decomposition of a positive definite NXN matrix looks like

$$\Sigma = \Gamma D \Gamma^T \tag{1}$$

where Γ is an orthogonal matrix of eigenvectors and D is a diagonal matrix of eigenvalues. Convince yourself that if instead of an $N \times N$ matrix, we had a kernel $K(s,t), s,t \in \mathcal{T}$, where K satisfies the hypotheses of the Mercer-Hilbert-Schmidt Theorem, then the Mercer-Hilbert-Schmidt Theorem is a generalization of the eigenvalue-eigenvector decomposition, generalizing from $1, 2, \cdots N$ to $\mathcal{T}$.

Hint: The elements $\sigma_{i,j}$ of Σ can be rewritten as

$$\sigma_{i,j} = \sum_{\nu=1}^{N} \lambda_\nu \gamma_\nu(i), \gamma_\nu(j) \tag{2}$$

where the λ_ν and the γ_ν are the eigenvalues and eigenvectors of Σ. Note: RN use ”characteristic values” for ”eigenvalues”