Statistics 860 hw2

1 Positive Definite Matrices

By Lecture 6: Show that the tensor (Kronecker) product of two positive definite matrices is positive definite. Show that the Schur product of two positive definite matrices is positive definite.

Let $A = \begin{pmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & & \vdots \\ a_{n1} & \cdots & a_{nn} \end{pmatrix}$, $B = \begin{pmatrix} b_{11} & \cdots & b_{1k} \\ \vdots & & \vdots \\ b_{k1} & \cdots & b_{kk} \end{pmatrix}$

Kronecker Product (tensor product) of A and B:
(The Kronecker Produce it a matrix of size $nk \times nk$)

$$A \otimes B = \begin{pmatrix} a_{11}B & \cdots & a_{1n}B \\ \vdots & & \vdots \\ a_{n1}B & \cdots & a_{nn}B \end{pmatrix}$$

Shur product of A and B: (A and B must have the same dimensions)

$$\begin{pmatrix} a_{11}b_{11} & \cdots & a_{1k}b_{1k} \\ \vdots & & \vdots \\ a_{k1}b_{k1} & \cdots & a_{kk}b_{kk} \end{pmatrix}$$