hw4 handed out Lect 5, due Lect 8

Let $\mathcal{H}$ be the collection of functions on $[0,1]$ with

$$f(0) = 0, f(t) = \int_0^t p(u)du$$

for some $p \in \mathcal{L}_2[0,1]$. That is, f and f' are in $\mathcal{L}_2$. This collection of functions can be shown to be an RKHS with the inner product $\langle f, g\rangle = \int_0^1 f'(u)g'(u)du$ and RK $min(s,t), \quad s,t \in [0,1]$).

Given y_1, y_2 and $\lambda > 0$, find a formula for $f_\lambda \in \mathcal{H}$ to minimize

$(y_1 - f(\frac{1}{2}))^2 + (y_2 - \int_0^1 f(u)du)^2 + \lambda \int_0^1 (f'(u))^2 du.$

Pages 24, 25 and 28 of lect5 are relevant.