Prelude to lect 17. Recall the next few slides, from lect 12, where we mentioned time and space models on the sphere, without talking about RKHS on the sphere. In lect 17 we discuss RK's on the sphere.

Read sphspl.pdf (Splines on the Sphere)
Read vecss.pdf (Vector Splines on the Sphere)

Time and Space Models on the Globe

Here $t = (t_1, t_2) = (x, P)$ where $x$ is year, and $P$ is (latitude, longitude). The RKHS of historical global temperature functions that was used in Chiang, Wahba, Johnson and Tribbia (1999) is

$$\mathcal{H} = [[1^{(1)}] \oplus [\phi] \oplus \mathcal{H}_s^{(1)}] \otimes [[1^{(2)}] \oplus \mathcal{H}_s^{(2)}],$$

a collection of functions $f(x, P)$, on $\mathcal{T} = \mathcal{T}^{(1)} \otimes \mathcal{T}^{(2)}$ $=\{1, 2, ..., 30\} \otimes \mathcal{S}$, where $\mathcal{S}$ is the sphere, and $\phi$ is a function which averages to 0 on $\mathcal{T}^{(1)}$. $\mathcal{H}$ and $f$ have the corresponding (six term) decompositions given next:

$$
\begin{array}{rcccccccc}
\mathcal{H} & = & [1] & \oplus & [\phi] & \oplus & [\mathcal{H}_s^{(1)}] & \oplus & [\mathcal{H}_s^{(2)}] \\
f(x,P) & = & C & + & d\phi(x) & + & f_1(x) & + & f_2(P) \\
 & = & mean & + & global & + & time & + & space \\
 & & & & time & & main & & main \\
 & & & & trend & & \textit{effect} & & \textit{effect}
\end{array}
$$

$$
\begin{array}{ccccc}
\oplus & & [[\phi] \otimes \mathcal{H}_s^{(2)}] & \oplus & [\mathcal{H}_s^{(1)} \otimes \mathcal{H}_s^{(2)}] \\
+ & & \phi(x) f_{\phi,2}(P) & + & f_{12}(x,P) \\
+ & & trend & + & space- \\
 & & by\ space & & time \\
 & & \textit{effect} & & interaction
\end{array}
$$

A sum of squares of second differences was applied to the time variable, and a spline on the sphere penalty (Wahba:1981,1982)) was applied to the space variable. For a cross country skier in the Midwest, as this author is, the results were very disappointing, in that they clearly showed a warming trend stretching from the Midwest towards Alaska (trend by space term) which was stronger than the global mean trend.

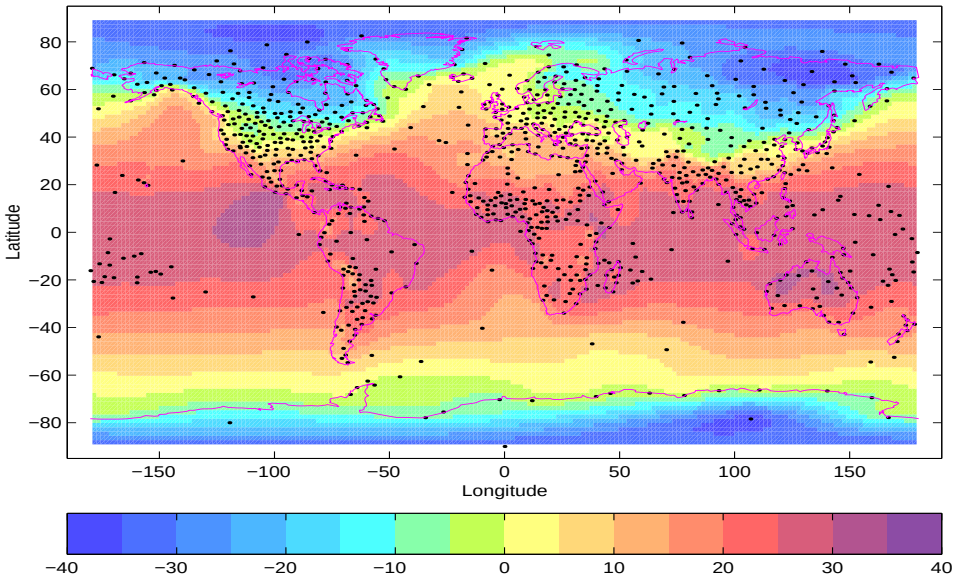


Figure 7: Mean of the historical average winter temperature ($^o$C), 1961-1990.

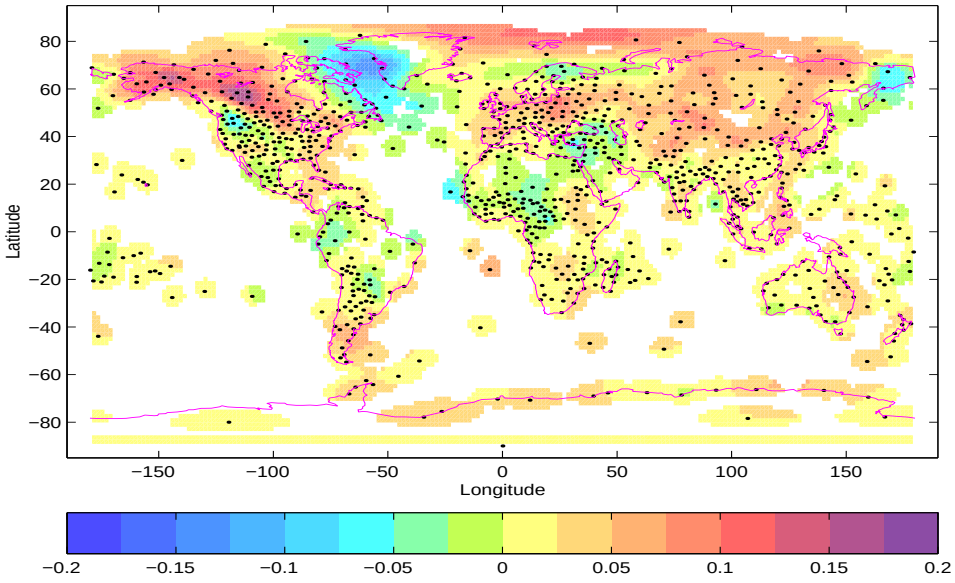


Figure 9: Linear trend of the historical average winter temperature ($^o$C/yr), 1961-1990.

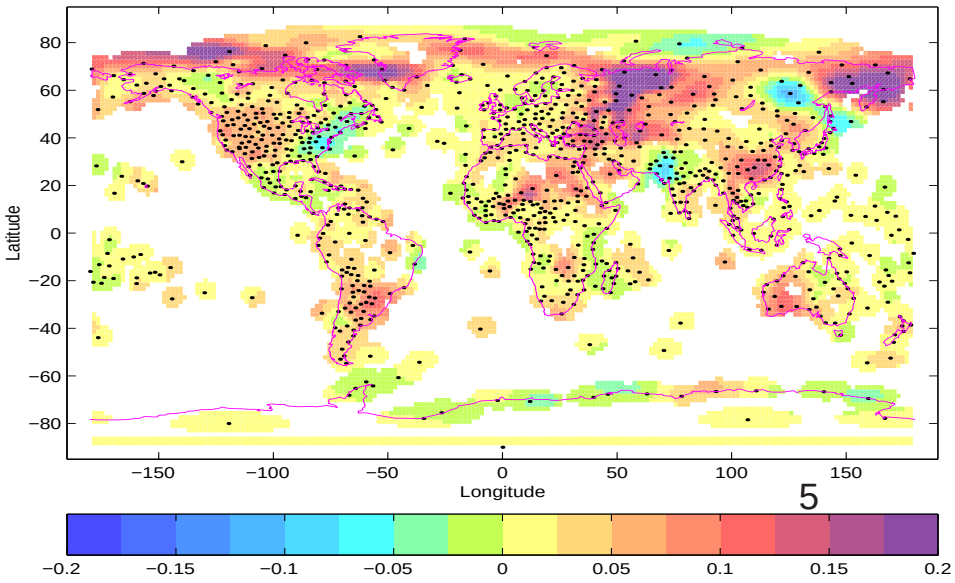


Figure 10: Linear trend of the GFDL forced minus background average winter temperature ($^o$C/yr),1961-1990.