Adaptive Tuning of Dynamical Systems Equations

Jianjian Gong, Grace Wahba (Statistics, Madison)
Donald R. Johnson (SSEC, Madison)
and Joseph Tribbia (NCAR)

Isaac Newton Institute, September 16, 1996

All TR's since late 93 up in
`http://www.stat.wisc.edu/~wahba`

e-mail `wahba@stat.wisc.edu`

Bibliography: See also
`http://www.stat.wisc.edu/~wahba`. Click on
TRLIST for (*)

Wahba, G., Johnson, D., Gao, F. & Gong, J. (1995), Adaptive tuning of nummerical weather prediction models: randomized GCV in three and four dimensional data assimilation, *Mon. Wea. Rev.* **123**, 3358–3369.

(*)Gong, J., Wahba, G., Johnson, D. & Tribbia, J. (1996), Adaptive tuning of numerical weather prediction models: simultaneous estimation of weighting, smoothing and physical parameters, University of Wisconsin-Madison Statistics Dept. TR 963, July 1996. Submitted.

Hutchinson, M. (1989), ‘A stochastic estimator for the trace of the influence matrix for Laplacian smoothing splines’, *Commun. Statist.-Simula.* **18**, 1059–1076.

Girard, D. (1991), ‘Asymptotic optimality of the fast randomized versions of GCV and C_L in ridge regression and regularization’, *Ann. Statist.* **19**, 1950–1963.

Synopsis

In Wahba, Johson, Gao and Gong (MWR 1995) we outlined how the randomized trace method could be used to adaptively tune numerical weather prediction models via generalized cross validation (GCV) and related methods. We have developed a 'toy' four dimensional data assimilation model (actually one space and one time variable), consisting of an equivalent barotropic vorticity equation on a latitude circle, and used it to demonstrate how this technique may be used to simultaneously tune weighting, smoothing and physical parameters.

Analyses both with the model as a strong constraint (corresponding to the usual 4D-Var approach) and as a weak constraint (corresponding theoretically to a fixed-interval Kalman smoother) are carried out. The conclusions are limited to the particular toy problem considered but it can be seen how more elaborate experiments could be carried out, as well as how the method might be applied in practice.

We have considered five adjustable parameters, two related to a distributed coefficient in the equivalent barotropic vorticity equation ('physical' parameters), one governing the relative weight given to observations *vs* forecast, one governing the relative weight given to observations *vs* goodness of fit to the model (in the weak constraint case) and one governing the damping of high frequency oscillations in the analysis at the final time point ('smoothing' parameter).

The weighting parameters and the smoothing parameter can, if desired, be interpreted as ratios of parameters in prior covariances.

Analyses are made with a low resolution model of the dynamics of the equivalent barotropic vorticity equation given noisy forecast (initial conditions) and noisy wind observations, and compared with nature evolved from exact initial conditions using a high resolution forward integration.

We found that these five (carefully chosen) parameters are simultaneously tunable on line, that is, simultaneously with the analysis, and that

1. The analysis is *equally* and strongly sensitive to both the choice of the observations *vs* forecast weighting parameter, and the choice of the smoothing parameter,

2. The analysis with the model as a weak constraint, based on the tuned estimate of the parameter governing how close the analysis satisfies the model, is somewhat better than the analysis using the model as a strong constraint, although estimation of this tuning parameter varies much more than the other parameters with replications of the experiment.

3. Good estimates of the physical parameters are obtained, however, these estimates are closer to those which make the imperfect model integrated forward (with perfect initial conditions) best fit nature than to the ‘true’ parameters in the nature run.

4. To some extent, changing the tuning and weighting parameters, cause the ‘optimal’ physical parameters to change, as a consequence, optimizing these parameters separately is suboptimal compared to optimizing them simultaneously.

5. The model error, that is, the difference between the evolved signal with the low resulution model and the high resolution model is correlated in both time and space, and furthermore depends on the signal, thus making it hard to describe ‘statistically’. This may partly explain why it is more difficult to tune the ‘strength’ of the model weak constraint.

RESULTS:
Demonstrated feasibility of simultaneously tuning physical, weighting and smoothing (bandwidth) parameters simultaneously, using GCV and UBR estimates, in a 'toy' problem. The experimental design and results suggest approaches for further study.

In practice:
Which parameters are important? (sensitivity)
Which can be tuned independently ?
Spatial variability?
Physics-based penalty functionals (J)?
Efficient design and search procedures?
Short or long-term adaptibility?
Modeling/detecting/estimating model error?

What did we learn, for this toy example ?

1. PMSE equally sensitive to the bandwidth (smoothing) parameter as to the parameter governing relative weight of observation and forecast.

2. $\widehat{U}_o$ is a good estimate of U_o which minimizes PMSE (which is not exactly the same as the 'NATURE' U_o).

3. Changing some parameters causes the 'optimal' value of other parameters to change.

4. Model is error correlated in time and space and depends on the signal. PMSE is relatively insensitive to this parameter near its minimum but the best weak constraint

found is marginally better than the strong constraint.

5. Demonstrates the feasibility of simultaneous tuning of a small number of heterogenous, but carefully selected parameters.