# References for G. Wahba Short Course

## PART I

**Reproducing Kernel Hilbert Spaces** 

- [Amo97] L. Amodei. Reproducing kernels of vector-valued function spaces. In A. LeMehaute, C. Rabut, and L. Schumaker, editors, *Surface Fitting and Multiresolution Methods*, pages 17–26, Nashville TN, 1997. Vanderbilt University Press.
- [Aro50] N. Aronszajn. Theory of reproducing kernels. *Trans. Am. Math. Soc.*, 68:337–404, 1950.
- [Par63] E. Parzen. Probability density functionals and reproducing kernel Hilbert spaces. In M. Rosenblatt, editor, *Proceedings of the Symposium on Time Series Analysis*, pages 155–169. Wiley, 1963.
- [Wah90] G. Wahba. Spline Models for Observational Data. SIAM, 1990. CBMS-NSF Regional Conference Series in Applied Mathematics, v. 59.
- [Wah92] G. Wahba. Multivariate function and operator estimation, based on smoothing splines and reproducing kernels. In M. Casdagli and S. Eubank, editors, *Nonlinear Modeling* and Forecasting, SFI Studies in the Sci of Complexity, Proc. Vol XII, pages 95–112. Addison-Wesley, 1992.
- [Wei82] H. Weinert, editor. Reproducing kernel Hilbert spaces: Application in signal processing. Hutchinson Ross, Stroudsburg, PA, 1982.

#### Moore-Aronszajn Theorem, Mercer-Hilbert-Schmitd Theorem

[Aro50] N. Aronszajn. Theory of reproducing kernels. *Trans. Am. Math. Soc.*, 68:337–404, 1950.

[RN55] F. Riesz and B. Sz. Nagy. *Functional Analysis*. Ungar, New York, 1955.

**Bernoulli Polynomials** 

[AS65] M. Abromowitz and I. Stegun. Handbook of Mathematical Functions with Formulas, Graphs and Mathematical Tables. U. S. Gov't. Printing Office, Washington, D.C., 1965.

The Representer Theorem

[KW71] G. Kimeldorf and G. Wahba. Some results on Tchebycheffian spline functions. J. Math. Anal. Applic., 33:82–95, 1971.

#### PART II

GCV, GML and Unbiased Risk for estimating smoothing parameters

[CW79] P. Craven and G. Wahba. Smoothing noisy data with spline functions: estimating the correct degree of smoothing by the method of generalized cross-validation. *Numer. Math.*, 31:377–403, 1979.

- [GW91] C. Gu and G. Wahba. Minimizing GCV/GML scores with multiple smoothing parameters via the Newton method. *SIAM J. Sci. Statist. Comput.*, 12:383–398, 1991.
- [Li85] K. C. Li. From Stein's unbiased risk estimates to the method of generalized cross-validation. Ann. Statist., 13:1352–1377, 1985.
- [Li86] K. C. Li. Asymptotic optimality of  $C_L$  and generalized cross validation in ridge regression with application to spline smoothing. *Ann. Statist.*, 14:1101–1112, 1986.
- [Li7b] K. C. Li. Asymptotic optimality for C sub p , C sub L , cross-validation and generalized cross validation: discrete index set. Ann. Math. Statist., 15:958–975, 1987b.
- [Mal73] C. Mallows. Some comments on  $C_p$ . Technometrics, 15:661–675, 1973.
- [Wah85] G. Wahba. A comparison of GCV and GML for choosing the smoothing parameter in the generalized spline smoothing problem. *Ann. Statist.*, 13:1378–1402, 1985.

The Thin Plate Spline

[Duc77] J. Duchon. Splines minimizing rotation-invariant seminorms in Sobolev spaces. In *Constructive Theory of Functions of Several Variables*, pages 85–100. Springer-Verlag, Berlin, 1977.

- [HG94] M. Hutchinson and P. Gessler. Splines more than just a smooth interpolator. *Geoderma*, 62:45–67, 1994.
- [WW80] G. Wahba and J. Wendelberger. Some new mathematical methods for variational objective analysis using splines and cross-validation. *Monthly Weather Review*, 108:1122– 1145, 1980.

'Distance' g(y, f)

- [HNP98] Xuming He, Pin Ng, and Stephen Portnoy. Bivariate quantile smoothing splines. *Journal of the Royal Statistical Society, Series B, Methodological*, 60:537–550, 1998.
- [Len77] Russell V. Lenth. Robust splines. *Communications in Statistics, Part A – Theory and Methods*, 6:847–854, 1977.
- [LLW00] Y. Lin, Y. Lee, and G. Wahba. Support vector machines for classification in nonstandard situations. Technical Report 1016, Department of Statistics, University of Wisconsin, Madison WI, 2000.
- [MN89] P. McCullagh and J. Nelder. *Generalized Linear Models*, *Second Edition*. Chapman and Hall, 1989.
- [O'S83] F. O'Sullivan. The analysis of some penalized likelihood estimation schemes. PhD thesis, Dept. of Statistics, University of Wisconsin, Madison, WI, 1983. Technical Report 726.
- [OYR86] F. O'Sullivan, B. Yandell, and W. Raynor. Automatic smoothing of regression functions in generalized linear models. J. Amer. Statist. Assoc., 81:96–103, 1986.

- [Utr81] Florencio I. Utreras. On computing robust splines and applications. *SIAM Journal on Scientific and Statistical Computing*, 2:153–163, 1981.
- [Wah69] G. Wahba. Estimating derivatives from outer space. Technical Report 989, Mathematics Research Center, 1969.
- [WLZ99] G. Wahba, Y. Lin, and H. Zhang. Generalized approximate cross validation for support vector machines, or, another way to look at margin-like quantities. Technical Report 1006, Department of Statistics, University of Wisconsin, Madison WI, 1999. to appear, Advances in Large Margin Classifiers, A. Smola, P. Bartlett, B. Scholkopf and D. Schurmans, eds, MIT Press.

### Integrals

- [Gir7c] D. Girard. Optimal regularized reconstruction in computerized tomography. *SIAM J. Sci. Statist. Comput.*, 8:934–950, 1987c.
- [NWGP84] D. Nychka, G. Wahba, S. Goldfarb, and T. Pugh. Crossvalidated spline methods for the estimation of three dimensional tumor size distributions from observations on two dimensional cross sections. J. Am. Stat. Assoc., 79:832–846, 1984.
- [O'S6a] F. O'Sullivan. A statistical perspective on ill-posed inverse problems. *Statistical Science*, 1:502–527, 1986a.

- [OW85] F. O'Sullivan and G. Wahba. A cross validated Bayesian retrieval algorithm for non-linear remote sensing. *J. Comput. Physics*, 59:441–455, 1985.
- [Wah85] G. Wahba. Design criteria and eigensequence plots for satellite- computed tomography. J. Atmos. Ocean. Tech., 2:125–132, 1985.

#### The Eta Theorem

[Par70] E. Parzen. Statistical inference on time series by rkhs methods. In R. Pyke, editor, *Proceedings 12th Biennial Seminar*, Montreal, 1970. Canadian Mathematical Congress. 1-37.

### The Histospline

- [DW82] N. Dyn and G. Wahba. On the estimation of functions of several variables from aggregated data. *SIAM J. Math. Anal.*, 13:134–152, 1982.
- [DWW79] N. Dyn, G. Wahba, and W. Wong. Comment on "Smooth pychnophylactic interpolation for geographical regions by W. Tobler. J. Am. Statist. Assoc., 74(367):530–535, 1979.
- [Wah81] G. Wahba. Numerical experiments with the thin plate histospline. *Commun. Statist.-Theor. Meth.*, A10:2475–2514, 1981.

- [Amo97] L. Amodei. Reproducing kernels of vector-valued function spaces. In A. LeMehaute, C. Rabut, and L. Schumaker, editors, *Surface Fitting and Multiresolution Methods*, pages 17–26, Nashville TN, 1997. Vanderbilt University Press.
- [BCL96] A. Bennett, B. Chua, and L. Leslie. Generalized inversion of a global weather prediction model. *Meteorology and Atmospheric Physics*, 60:165–178, 1996.
- [KS85] C. Kravaris and J.H. Seinfeld. Identification of parameters in distributed parameter systems by regularization. SIAM J. Control Opt., 23:217–241, 1985.
- [Utr85] F. Utreras. Smoothing noisy data under monotonicity constraints, existence, characterization and convergence rates. *Numer. Math.*, 47:611–625, 1985.
- [VW87] M. Villalobos and G. Wahba. Inequality constrained multivariate smoothing splines with application to the estimation of posterior probabilities. J. Am. Statist. Assoc., 82:239– 248, 1987.
- [Wah82] G. Wahba. Constrained regularization for ill posed linear operator equations, with applications in meteorology and medicine. In S. Gupta and J. Berger, editors, *Statistical Decision Theory and Related Topics, III, Vol.2*, pages 383– 418. Academic Press, 1982.
- [Wah99] G. Wahba. Adaptive tuning, four dimensional variational data assimilation and representers in rkhs. In ECMWF,

editor, *Diagnosis of Data Assimilation Systems*, pages 45–52, Reading England, 1999. European Center for Medium Range Weather Prediction.

# PART III

SS-ANOVA

[BR98]	B. Brumback and J. Rice. Smoothing spline models for the analysis of nested and crossed samples of curves. <i>J.</i> <i>Amer. Statist. Assoc.</i> , 93:961–991, 1998.
[CGW89]	Z. Chen, C. Gu, and G. Wahba. Comments to 'Linear Smoothers and Additive Models', by Buja, Hastie and Tibshirani. <i>Ann. Statist.</i> , 17:515–521, 1989.
[GW91]	C. Gu and G. Wahba. Comments to 'Multivariate Adap- tive Regression Splines', by J. Friedman. <i>Ann. Statist.</i> , 19:115–123, 1991.
[GW93a]	C. Gu and G. Wahba. Semiparametric analysis of vari- ance with tensor product thin plate splines. <i>J. Royal</i> <i>Statistical Soc. Ser. B</i> , 55:353–368, 1993.
[GW93b]	C. Gu and G. Wahba. Smoothing spline ANOVA with component-wise Bayesian "confidence intervals". <i>J. Computational and Graphical Statistics</i> , 2:97–117, 1993.

[GWKK99] F. Gao, G. Wahba, R. Klein, and B. Klein. Smoothing spline ANOVA for multivariate Bernoulli observations,

with applications to ophthalmology data. Technical Report 1009, Department of Statistics, University of Wisconsin, Madison WI, 1999.

- [VCKW99] A. Verbyla, B. Cullis, M. Kenward, and S. Welham. The analysis of designed experiments and longitudinal data using smoothing splines. J. Roy. Stat. Soc. C, 48:269– 311, 1999.
- [Wan97] Y. Wang. GRKPACK: Fitting smoothing spline analysis of variance models to data from exponential families. *Commun. Statist. Simulation and Computation*, 26:765–782, 1997.
- [Wan98a] Y. Wang. Mixed-effects smoothing spline ANOVA. J. Roy. Stat. Soc. B, 60:159–174, 1998.
- [Wan98b] Y. Wang. Smoothing spline models with correlated random errors. J. Amer. Statist. Assoc., 93:34–348, 1998.
- [Wan98c] Y. Wang. Smoothing spline models with correlated random errors. J. Amer. Statist. Assoc., 93:341–348, 1998.
- [WB96] Y. Wang and M. Brown. A flexible model for human circadian rhythms. *Biometrics*, 52:588–596, 1996.
- [WW98] Y. Wang and G. Wahba. Comments to 'Smoothing spline models for the analysis of nested and crossed samples of curves' by B. Brumback and J. Rice. J. Amer. Statist. Assoc., 93:976–980, 1998.
- [WWG<sup>+</sup>94] G. Wahba, Y. Wang, C. Gu, R. Klein, and B. Klein. Structured machine learning for 'soft' classification with smoothing spline ANOVA and stacked tuning, testing and evaluation. In J. Cowan, G. Tesauro, and

J. Alspector, editors, *Advances in Neural Information Processing Systems 6*, pages 415–422. Morgan Kauffman, 1994.

- [WWG<sup>+</sup>95] G. Wahba, Y. Wang, C. Gu, R. Klein, and B. Klein. Smoothing spline ANOVA for exponential families, with application to the Wisconsin Epidemiological Study of Diabetic Retinopathy. *Ann. Statist.*, 23:1865– 1895, 1995. Neyman Lecture.
- [WWG<sup>+</sup>97] Y. Wang, G. Wahba, C. Gu, R. Klein, and B. Klein. Using smoothing spline ANOVA to examine the relation of risk factors to the incidence and progression of diabetic retinopathy. *Statistics in Medicine*, 16:1357– 1376, 1997.

## **Tuning Non-Gaussian Models**

- [Gu92] C. Gu. Cross-validating non-Gaussian data. J. Comput. Graph. Stats., 1:169–179, 1992.
- [GWJT98] J. Gong, G. Wahba, D. Johnson, and J. Tribbia. Adaptive tuning of numerical weather prediction models: simultaneous estimation of weighting, smoothing and physical parameters. *Monthly Weather Review*, 125:210–231, 1998.
- [LWX<sup>+</sup>98] X. Lin, G. Wahba, D. Xiang, F. Gao, R. Klein, and B. Klein. Smoothing spline ANOVA models for large data sets with Bernoulli observations and the randomized GACV. Technical Report 998, Department of

Statistics, University of Wisconsin, Madison WI, tent. acc. Ann. Statist., 1998.

- [WJGG95] G. Wahba, D. Johnson, F. Gao, and J. Gong. Adaptive tuning of numerical weather prediction models: randomized GCV in three and four dimensional data assimilation. *Mon. Wea. Rev.*, 123:3358–3369, 1995.
- [WLG<sup>+</sup>99] G. Wahba, X. Lin, F. Gao, D. Xiang, R. Klein, and B. Klein. The bias-variance tradeoff and the randomized GACV. In M. Kearns, S. Solla, and D. Cohn, editors, *Advances in Information Processing Systems 11*, pages 620–626. MIT Press, 1999.
- [XW96] D. Xiang and G. Wahba. A generalized approximate cross validation for smoothing splines with non-Gaussian data. *Statistica Sinica*, 6:675–692, 1996.

**Randomized Trace Techniques** 

- [Gao99] F. Gao. Iterated ranGACV: a computational proxy for the comparative Kullback-Leibler distance. Technical Report 1011, Department of Statistics, University of Wisconsin, Madison WI, 1999.
- [Gir89] D. Girard. A fast 'Monte-Carlo cross-validation' procedure for large least squares problems with noisy data. *Numer. Math.*, 56:1–23, 1989.
- [Gir91] D. Girard. Asymptotic optimality of the fast randomized versions of GCV and  $C_L$  in ridge regression and regularization. Ann. Statist., 19:1950–1963, 1991.

- [Gir98] D. Girard. Asymptotic comparison of (partial) crossvalidation, GCV and randomized GCV in nonparametric regression. *Ann. Statist.*, 126:315–334, 1998.
- [Hut89] M. Hutchinson. A stochastic estimator for the trace of the influence matrix for Laplacian smoothing splines. *Commun. Statist.-Simula.*, 18:1059–1076, 1989.

**Bayesian 'Confidence Intervals'** 

- [GW93] C. Gu and G. Wahba. Smoothing spline ANOVA with component-wise Bayesian "confidence intervals". J. Computational and Graphical Statistics, 2:97–117, 1993.
- [Wah83] G. Wahba. Bayesian "confidence intervals" for the crossvalidated smoothing spline. J. Roy. Stat. Soc. Ser. B, 45:133–150, 1983.
- [WW95] Y. Wang and G. Wahba. Bootstrap confidence intervals for smoothing splines and their comparison to Bayesian 'confidence intervals'. J. Statist. Comput. Simul., 51:263–279, 1995.