Stat/For/Hort 571 — Final Exam, Fall 2000 — Partial Solutions

1. (a) $H_0 : \mu_A = \mu_B = \mu_C = \mu_D = \mu_E$. SSError $= 6 \times 4.51^2 + 4 \times 4.76^2 + 3 \times 5.59^2 + 4 \times 4.44^2 + 5 \times 5.14^2 = 517.37$. The ANOVA table looks like:

Source	df	SS	MS
Trt	4	567.38	141.85
Error	22	517.37	23.52
Total	26	1084.75	

Thus $F = 141.85/23.52 = 6.03$ on (4,22) df. $0.001 <$ p-value < 0.005. There is very strong evidence against H_0. Note: $s_p = \sqrt{23.52}$.

(b) $H_0 : \frac{1}{2}(\mu_B + \mu_D) = \frac{1}{2}(\mu_A + \mu_E)$. Hence the contrast of interest is $\frac{1}{2}(\bar{y}_B - \bar{y}_D) - \frac{1}{2}(\bar{y}_A + \bar{y}_E) = -4.48$ with standard error $s_p\sqrt{\frac{(-1/2)^2}{7} + \frac{(1/2)^2}{5} + \frac{0}{4} + \frac{(1/2)^2}{5} + \frac{(-1/2)^2}{6}} = \sqrt{23.52} \times 0.42 = 2.04$. Thus $t = -4.48/2.04 = -2.19$ on 22 df. $0.02 <$ p-value < 0.05. There is moderate evidence again H_0.

2. (a) $\hat{b}_1 = \frac{\sum x_i y_i - \frac{1}{n}(\sum x_i)(\sum y_i)}{\sum x_i^2 - \frac{1}{n}(\sum x_i)^2} = 1.104$ and $\hat{b}_0 = \bar{y} - \hat{b}_1\bar{x} = -1.245$.

(b) $H_0 : b_1 = 1.3$. SSTotal $= \sum y_i^2 - \frac{1}{n}(\sum y_i)^2 = 1.40$. The ANOVA table for regression looks like:

Source	df	SS	MS
Regression	1	1.14	1.14
Error	9	0.26	0.029
Total	10	1.40	

Also $\sum x_i^2 - \frac{1}{11}(\sum x_i)^2 = 0.94$. So, we have $t = \frac{1.104-1.3}{\sqrt{0.029/0.94}} = -1.12$ on 9 df. Thus p-value > 0.10. There is no evidence against the claim that the slope is 1.3.

(c) We have $\hat{y}_{est} = -1.245 + 1.104 \times 1.2 = 0.0798$ with $\text{se}(\hat{y}_{est}) = \sqrt{0.029(\frac{1}{11} + \frac{(1.2-0.82)^2}{0.94})} = 0.084$. The 95% confidence interval for the expected skeletal mass is therefore: $0.0798 \pm 0.084 \times t_{9,.025}$ or $-0.11 < \hat{y}_{est}(1.2) < 0.27$.

3. The "anticipated" 95% confidence interval for $\mu_A - \mu_B$ is $(\bar{y}_A - \bar{y}_B) \pm z_{.025}\sigma\sqrt{\frac{1}{8} + \frac{1}{n}}$. Set its half width to $2.70/2 = 1.35$. Thus, $1.35 = 1.96 \times 1.8 \times \sqrt{\frac{1}{8} + \frac{1}{n}}$ and solve for n. $n = 46.68$ and is rounded up to 47.

4. $H_0 : p = 0.5$ for each comparison. Comparison-wise: for $Y_{red} \sim B(6,p)$, p-value $= 2 \times P(Y_{red} = 6) = 0.031$; for $Y_{blue} \sim B(9,p)$, p-value $= 2 \times [P(Y_{blue} = 8) + P(Y_{blue} = 9)] = 0.039$; for $Y_{green} \sim B(40,p)$, $Y_{NA} \sim N(20,10)$ under H_0 and p-value $= 2 \times P(Y_{NA} \leq 12) = 2 \times P(Z \leq -2.53) = 0.011$. Experiment-wise: p-values are 3 times the comparison-wise p-values. The results are $0.093, 0.117, 0.033$ for the red, blue, and green coin. Hence reject H_0 at the 5% level only for the green coin.

5. $H_0 : \mu = 1.5$ vs. $H_A : \mu \neq 1.5$. Let $Y =$ disease score. By CLT, $Z = (\bar{Y} - 1.5)/s_{\bar{Y}} \sim N(0,1)$ under $H_0 : \mu = 1.5$. For the given data, $\bar{y} = \frac{1}{200}(70 \times 1 + 80 \times 2 + 50 \times 3) = 1.9$ and $s^2 = \frac{1}{199}[70 \times (1 - 1.9)^2 + 80 \times (2 - 1.9)^2 + 50 \times (3 - 1.9)^2] = 0.59$. Then $z = (1.9 - 1.5)/\sqrt{0.59/200} = 7.35$ and p-value < 0.001. Hence, there is very strong evidence against H_0.