Stat/For/Hort 571
Final Exam, Fall 2002
Brief Solutions

1. (a) $F = 32.65/3.4678 = 9.42$. Checking the F-table with degrees of freedom 4 and 10, we see the p-value is less than 0.05.

Source	df	SS	MS
Treatments	4	130.600	32.65
Error	10	34.678	3.4678
Total	14	165.278	

(b) The null hypothesis is
H_0: $\mu_A = \mu_B = \mu_C = \mu_D = \mu_E$.

(c)

$$\begin{aligned} T &= \frac{\frac{11.26+13.54}{2} - \frac{14.06+19.44+17.68}{3}}{\sqrt{3.4678}\sqrt{\frac{1/4+1/4+1/9+1/9+1/9}{3}}} \\ &= \frac{-4.635}{0.9815} \\ &= -4.72 \end{aligned}$$

p-value < 0.002. Reject the null hypothesis.

(d) $LSD = 2.228\sqrt{3.678}\sqrt{2/3} = 3.388$. The display is

<u>11.26 13.59 14.06</u> <u>17.68 19.44</u>

2. (a) $n = 9$. Plugging in the formula for the least square estimates, we get $\hat{b}_1 = 0.714$, and $\hat{b}_0 = \bar{y} - \hat{b}_1\bar{x} = 1.148$.

(b)

Source	df	SS	MS
Regression	1	271.32	271.32
Error	7	164.68	23.53
Total	8	436	

(c)

$$\begin{aligned} T &= \frac{1.148 - 10}{\sqrt{23.53}\sqrt{\frac{1}{9} + \frac{\bar{x}^2}{\sum(x_i - \bar{x})^2}}} \\ &= -2.15 \end{aligned}$$

The p-value is between 0.05 and 0.10.

(d) $\hat{y}_{pred} = 1.148 + 0.714 \times 13 = 10.43$. The 90% confidence interval for the prediction is

$$\begin{aligned} & 10.43 \pm 1.895\sqrt{23.53}\sqrt{1 + \frac{1}{9} + \frac{(13-18)^2}{\sum(x_i - \bar{x})^2}} \\ =\ & 10.43 \pm 9.89. \end{aligned}$$

3. (a)

$$\begin{aligned} Z &= \frac{25 - 29}{\sqrt{\frac{40}{32} + \frac{16}{8}}} \\ &= -2.21 \end{aligned}$$

p-value $= 0.0272$. Reject the null hypothesis at 0.05 level.

(b)

$$\begin{aligned} Z &= \frac{26 - 29}{\sqrt{\frac{16}{8}}} \\ &= -2.12 \end{aligned}$$

p-value $= 0.034$. Reject the null hypothesis at 0.05 level.

4. For Rule A: $\bar{X} \sim N(62, 800/100)$, and the power is

$$\begin{aligned} & P(\bar{X} > 60) \\ =\ & P(Z > \frac{60 - 62}{\sqrt{8}}) \\ =\ & P(Z > -0.71) \\ =\ & 0.7611. \end{aligned}$$

For Rule B: we have

$$\begin{aligned} & P(X_i > 70) \\ =\ & P((Z > \frac{70 - 62}{\sqrt{800}}) \\ =\ & P(Z > 0.28) \\ =\ & 0.3817. \end{aligned}$$

Denote the number of defective observations as W. Then $W \sim B(100, 0.3817) \approx N(38.97, 23.78)$, and the power is

$$\begin{aligned} & P(W > 36) \\ =\ & P(Z > \frac{36 - 38.97}{\sqrt{23.78}}) \\ =\ & P(Z > -0.61) \\ =\ & 0.7291. \end{aligned}$$

Therefore Rule A is more powerful.

5. Under the null hypothesis, the data come from a standard normal distribution, and the probability of the three categories are $P(Z < -1.2) = 0.1151$, $P(-1.2 < Z < 1.4) = 0.8041$, and $P(Z > 1.4) = 0.0808$. Therefore the expected counts are $120 \times 0.1151 = 13.81$, $120 \times 0.8041 = 96.49$ and $120 \times 0.0808 = 9.70$, respectively.

$$\begin{aligned} X^2 &= \frac{(24 - 13.81)^2}{13.81} + \frac{(85 - 96.49)^2}{96.49} + \frac{(11 - 9.7)^2}{9.7} \\ &= 9.06 \end{aligned}$$

on 2 degrees of freedom. The p-value is between 0.01 and 0.025.