

Midterm I

Name: _____

For the section that you *attend* please indicate:**Instructor:**(circle one) Chuang Nordheim**TA:** (circle one) Cong Li Shen

Instructions:

1. This exam is open book. You may use textbooks, notebooks, class notes, and a calculator.
2. Do all your work in the spaces provided. If you need additional space, use the back of the preceding page, indicating *clearly* that you have done so.
3. To get full credit, you must show your work. Partial credit will be awarded.
4. Some partial computations have been provided on some questions. You may find some *but not necessarily all* of these computations useful. You may assume that these computations are correct.
5. Do not dwell too long on any one question. Answer as many questions as you can.
6. Note that some questions have multiple parts. For some questions, these parts are independent, and so you can work on part (b) or (c) separately from part (a).

 For graders' use:

Question	Possible Points	Score
1	20	
2	20	
3	20	
4	20	
5	20	
Total	100	

1. The height of 6-year old poplar seedlings is known to be approximately normally distributed. It is known that the variance of seedling height is 3800 cm^2 . A researcher has developed a new variety of poplar seedlings and wishes to test the null hypothesis that the mean height (of 6-year old seedlings) of this variety is 420 cm (versus the two-sided alternative). From a large experimental planting, 19 seedlings are randomly selected and their heights measured. The heights (in cm) are:

423 504 384 483
 364 477 543 561
 411 418 491 402
 520 335 399 459
 468 430 476

Let y_i be the height of the i th seedling. The following information might be of value:

$$\begin{aligned}\sum y_i &= 8548 & \sum y_i^2 &= 3912622 \\ n \sum y_i &= 162412 & n \sum y_i^2 &= 74339818\end{aligned}$$

- (a) Make a useful display of these data. Comment on the display.
 - (b) Carry out a test of the null hypothesis. Find and interpret the p -value. Do you reject the null hypothesis at 10%? at 5%? at 1%?
2. (a) Let $Y \sim N(\mu, \sigma^2)$. Find the probability that Y will fall more than two but less than three standard deviations away from the mean.
 - (b) Let W be the weight of a random adult female from a certain breed of pigs. Suppose it is known that W is normally distributed with mean 85 kg and variance 60 kg². Let \bar{W} be the mean of a random sample of 16 pigs. Find the value w_* so that the probability that \bar{W} is less than w_* is 0.75.
3. Let Y be the number of young calves with a particular bacterial infection. Assume that the usual binomial assumptions hold for Y . Suppose it is known that the probability for a randomly selected calf to have the infection is 0.3.
 - (a) If a random sample of 8 calves is selected, what is the probability that the number of calves with the infection is 1 or less?
 - (b) If a random sample of 80 calves is selected, what is the probability that the number of calves with the infection is 10 or less?
 - (c) If your answers to (a) and (b) are the same, explain why. If the answers differ, explain why.
 4. The discrete random variable Y can take on the values 0, 1, and 5. The probabilities for each value are as follows: $P(Y = 0) = 0.2$, $P(Y = 1) = 0.5$, and $P(Y = 5) = 0.3$. (If you would like to view Y as something “realistic” think of it as representing the number of dollars you could win in a carnival game.)
 - (a) Find the mean and variance of Y .
 - (b) Consider a random sample of size n from this distribution. Define the random variable \bar{Y} as $\bar{Y} = \frac{1}{n} \sum_{i=1}^n Y_i$; that is, \bar{Y} is the sample mean. If $n = 100$, find $P(\bar{Y} < 1.5)$.
 - (c) What did you need to assume to make the computation in (b)?
 5. Let $Y \sim N(\mu, \sigma^2)$. Consider a random sample of size n . We know that $V^2 = (n-1)S^2/\sigma^2$ is distributed as a chi-squared random variable on $(n-1)$ degrees of freedom. It is known that $E(V^2) = n-1$ and $\text{Var}(V^2) = 2(n-1)$; that is, the mean of V^2 is $n-1$ and the variance of V^2 is $2(n-1)$. (These facts are given on p. 118 of the Course Notes.)
 - (a) Determine $E(V^2)$, $\text{Var}(V^2)$ and the standard deviation of V^2 if $n = 4$.
 - (b) For the V^2 corresponding to a random sample of size 4 ($n = 4$), find the probability (to the precision possible with the tables in the Course Notes) that V^2 will fall within one standard deviation of its mean. (Hint: The distribution of V^2 is not symmetrical.)