Midterm I

Name:_____

For the section that you *attend* please indicate:
Instructor:(circle one) Nordheim Rasmussen

TA: (circle one) Cheng Sego Song

Instructions:

1. This exam is open book. You may use textbooks, notebooks, class notes, and a calculator.
2. Do all your work in the spaces provided. If you need additional space, use the back of the preceding page, indicating *clearly* that you have done so.
3. To get full credit, you must show your work. Partial credit will be awarded.
4. Some partial computations have been provided on some questions. You may find some *but not necessarily all* of these computations useful. You may assume that these computations are correct.
5. Do not dwell too long on any one question. Answer as many questions as you can.
6. Note that some questions have multiple parts. For some questions, these parts are independent; in such cases you can work, for example, on part (b) or (c) separately from part (a).

For graders' use:

Question	Possible Points	Score
1	20	
2	16	
3	24	
4	20	
5	20	
Total	100	

1. A soil scientist studies the thickness of the top layer of soil in a certain geological region. The scientist obtains a random sample of 15 thickness readings; the following are the data in cm.

   9.9 11.5 12.8 9.1 11.6 12.1 10.9 12.6 10.9 8.3 10.5 8.0 10.3 13.2 11.9

   Let y_i be the thickness of the ith reading. The following information might be of value.

$$n\sum y_i = 2454.0 \qquad \sum y_i^2 = 1819.34$$

$$(\sum y_i^2)/n = 121.289 \qquad \sum y_i = 163.6$$

(a) Make a useful display of these data; comment on the display.

(b) Find the sample median, mean, and variance for these data.

(c) Let q be the value of the sample variance calculated in part (b) of this question. Let Y be the random variable representing the thickness of a soil reading; assume that $Y \sim N(11, 4.0)$. Let S^2 be the random variable representing the sample variance from this distribution for a sample of size 15. Find $P(S^2 < q)$.

2. Let Y be the height of 5 year-old pin oak seedlings. Assume that Y is approximately distributed as a normal with a known variance of 0.10 m^2. It is claimed that the mean height of such oak seedlings is 1.30 m (with the alternative being two-sided). A random sample of 8 seedlings was taken and the following heights were obtained:

   1.25 1.10 .65 1.30 1.55 .90 1.10 .85

   Carry out a test of the null hypothesis. Formally state the hypotheses; compute and interpret the p-value. Are the results significant at 10%? at 5%? at 1%?

3. It is known that the fat content of milk (in percent) from dairy cows of a given breed is distributed approximately normally with a mean of 3.62 and a variance of 0.025. Consider taking a random sample of n cows from this breed. Let $\bar{Y}$ be the mean (a random variable) for this sample.

   (a) If $n = 18$, find the value of y_* so that there is a probability of 0.80 that $\bar{Y}$ is less than y_*.

   (b) What sample size, n, is needed so that the width of the interval in which the central 95% of the distribution of $\bar{Y}$ falls is 0.06. [[Hint: If l_* and u_* are the lower and upper ends of this interval, $P(l_* \leq \bar{Y} \leq u_*) = 0.95$ with a probability of 0.025 outside of the interval on each end. Draw a picture!!]]

4. I have two cats named Mittens and Frisky and a dog named Butch. All three of the animals enjoy spending time in my living room. There is a probability of 0.6 that Mittens is in my living room; similarly there are probabilities of 0.4 and 0.2 for Frisky and Butch respectively to be in my living room. The two cats behave independently of each other. However, Butch will never be in the living room if either or both cats are present. Similarly, neither cat will be in the living room if Butch is there.

   (a) Find the probability that Frisky is in the living room given that Mittens is in the living room.

   (b) Find the probability that Frisky is in the living room given that Butch is in the living room.

   (c) Find the probability that only Frisky is in the living room.

   (d) Find the expected number of animals in my living room.

5. Let Y be the number of potatoes with a certain type of deformity.

   (a) Assume $Y \sim B(12, 0.2)$. Find the mean, μ, and variance, σ^2, of Y.

   (b) Consider the distribution given in part (a). Let $q = \mu - \sigma$. Find Prob$(Y < q)$.

   (c) Suppose n is a number larger than 100. Let $Y \sim B(n, 0.2)$. Define q by $q = \mu - \sigma$ where μ and σ correspond to this binomial. Find (approximately) Prob$(Y < q)$. [[Hint: Draw a picture!!]]