Midterm II

Instructions:

1. This exam is open book. You may use textbooks, notebooks, class notes, and a calculator.
2. Do all your work in the spaces provided. If you need additional space, use the back of the preceding page, indicating *clearly* that you have done so.
3. To get full credit, you must show your work. Partial credit will be awarded.
4. Do not dwell too long on any one question. Answer as many questions as you can.
5. Note that some questions have multiple parts. For some questions, these parts are independent; in such cases you can work, for example, on part (b) or (c) separately from part (a).

For graders' use:

Question	Possible Points	Score
1	18	
2	20	
3	22	
4	22	
5	18	
Total	100	

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1. A study was conducted to compare two different exercise programs in terms of weight loss in adult men. Twelve men were available for this study. These men were divided into six groups of 2 men so that the two men in each group were similar in age and initial weight. One man in each group was assigned to each exercise program. The weight losses (in pounds) for all 12 men were recorded after six weeks.

Group	1	2	3	4	5	6
Program A	7	4	11	8	3	12
Program B	10	9	11	7	6	13

   (a) Perform a test of the null hypothesis of equal efficacy for the two exercise programs versus the two-sided alternative. Interpret the results.

   (b) State the assumptions underlying the test you use in part (a). [Note – You do *not* need to justify these assumptions.]

2. (a) A study was undertaken to compare the coefficients of digestibility of dry matter for four (4) diets fed to goats. Five (5) randomly selected goats were assigned to each treatment. The sample means and standard deviations are as follows:

Diet	1	2	3	4
Sample mean	55.24	52.96	61.06	56.40
Sample standard dev	4.01	4.48	3.99	3.02

   Also, $SSTRT = 174.47$. Compute the ANOVA table. Carry out the test of the null hypothesis that the mean coefficients of digestibility are the same for all 4 diets. Interpret the results.

(b) Suppose now that diets 1 and 2 had each been given to 8 goats and diets 3 and 4 had each been given to 2 goats. Suppose that the sample means and standard deviations are exactly the same as given above!! This results in $SSTR = 111.11$. Compute the ANOVA table and perform the same test as in part (a).

(c) If the conclusions are the same in parts (a) and (b), explain why. If they are different, explain why.

3. We know that for inference for μ with underlying normal data, there is an exact correspondence between testing and confidence intervals (CI's) in that, if a test for $H_0 : \mu = c$ versus the two-sided alternative is rejected at level α, a $(1 - \alpha)$ CI for μ will include c. We also know that this correspondence is *not* exact for inference with binomial data (using the normal approximation). The purpose of this question is to explore this correspondence in inference for σ^2 with underlying normal data. Parts (a), (b), and (c) will (hopefully) provide you with some insight; you will give your conclusion in part (d).

   Suppose that a random sample of size 9 is obtained from $N(\mu, \sigma^2)$ and $s^2 = 15.0$.

   (a) Find a 95% CI for σ^2.

   (b) Test $H_0 : \sigma^2 = 6.8$ versus the two-sided alternative.

   (c) Test $H_0 : \sigma^2 = 56.0$ versus the two-sided alternative.

   (d) Do you believe that the correspondence between testing and CI's is exact? Give your reasoning.

4. For each of the questions below, the *italicized* statement is either True or False. Indicate whether the statement is True or False and provide a justification for your response.

   (a) Let $Y_1 \sim N(\mu_1, \sigma_1^2)$ and let $Y_2 \sim N(\mu_2, \sigma_2^2)$. A random sample of 15 observations is obtained from Y_1 with $\bar{y}_1 = 28.6$ and $s_1^2 = 10.2$. A random sample of 6 observations is obtained from Y_2 with $\bar{y}_2 = 80.4$ and $s_2^2 = 3.2$. *The 95% confidence interval (CI) for μ_2 (calculated from the data on Y_2 only) is wider than the 95% CI for μ_1 (calculated from the data on Y_1 only).*

   (b) Let $Y \sim N(\mu, \sigma^2)$ and you are interested in testing $H_0 : \mu = 25$ versus $H_0 : \mu > 25$. The test you plan to perform is based on 4 independent trials. In each trial, a random sample of size $n = 10$ is selected and a sample mean is computed. Suppose you set the rejection criterion to be:

   $$\text{reject } H_0 \text{ if } \bar{Y} > 25.0 \text{ for } \textbf{all } 4 \text{ trials.}$$

   This test has a Type I error rate (α) that is less than 0.05.

   (c) Let $Y \sim N(\mu, \sigma^2)$. You are interested in testing $H_0 : \mu = 25$ versus $H_0 : \mu > 25$. You plan on conducting 4 independent trials, each with n=10. Suppose you set the criterion that you will reject H_0 if $\bar{Y} > 25.0$ for each of the 4 trials. *This test has a Type I error rate (α) that is less than 0.05.*

5. A standard procedure for inserting a gene in soy bean plants has a success rate of 0.75. A new procedure has been developed with the expectation that it will have a higher success rate. (That is, it is not expected that the rate could be lower.) A random sample of n soy bean seedlings will be selected and the null hypothesis that the success rate is equal to 0.75 will be rejected if the observed proportion of plants with successful insertion is 0.81 or higher. Find n so that the power is 0.90 if the true success rate is 0.85.