Stat/For/Hort 571
Midterm II, Fall 2002
Brief Solutions

1. (a)

Source	df	SS	MS	F
Insecticide	4	38.766	9.692	2.4424
Error	16	63.490	3.968	
Total	20	102.256		

p-value is in $(0.05, 0.1)$. At 0.05 level, reject the null hypothesis that the population mean wing lengths corresponding to the 5 species are all equal.

(b) The 95% confidence interval is $(63.49/28.85, 63.49/6.91) = (2.2, 9.2)$.

2. Let the number of wasps turning to the right be X. Under the null hypothesis H_0 we have $X \sim B(11, 0.65)$. Therefore,

$$\begin{aligned}
\alpha &= P(\text{rejecting the null hypothesis}|p = 0.65) \\
&= P(X = 0, 1, \text{or} 11|p = 0.65) \\
&= 0.65^0(1-0.65)^{11} + 11 \times 0.65^1(1-0.65)^{10} + 0.65^{11} \\
&= 0.00001 + 0.00875 + 0.000197 \\
&= 0.0089.
\end{aligned}$$

3. (a) We have

$$\begin{aligned}
vr_1 &= s_1^2/n_1 = 4.9167/4 = 1.23 \\
vr_2 &= s_2^2/n_2 = 1195.982/8 = 149.5
\end{aligned}$$

$$\begin{aligned}
adf &= \frac{(vr_1 + vr_2)^2}{(vr_1^2/(n_1-1)) + (vr_2^2/(n_2-1))} \\
&= 7.11.
\end{aligned}$$

Round down to $adf = 7$.

$$p - value = 2 \times (0.005, 0.01) = (0.01, 0.02).$$

(b) Use the Mann-Whitney test. $n_1 = 4$, $n_2 = 8$, $T^* = 12$, $T^{**} = n_1(n_1 + n_2 + 1) - T^* = 40$. Therefore $T = \min(T^*, T^{**}) = 12$. Now the table give p-value $\in (0.01, 0.05)$.

(c) (b) is more appropriate. A stem-and-leaf plot reveals that the normality assumption is not likely to be valid for this data. (a) requires normality, (b) does not.

4. (a) $\hat{p}_1 = 35/60 = 0.58$, $\hat{p}_2 = 30/75 = 0.40$. Therefore the 90% confidence interval for $p_1 - p_2$ is

$$\hat{p}_1 - \hat{p}_2 \pm z_{0.05}\sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{60} + \frac{\hat{p}_2(1-\hat{p}_2)}{75}} = 0.18 \pm 0.14 = (0.04, 0.32).$$

(b) The assumptions are $n_1\hat{p}_1 \geq 5$, $n_1(1-\hat{p}_1) \geq 5$, $n_2\hat{p}_2 \geq 5$, $n_2(1-\hat{p}_2) \geq 5$. These are easily checked to be satisfied.

5. Under H_0, we have $E(X) = 0 \times 1/3 + 2 \times 1/3 + 7 \times 1/3 = 3$; $\text{var}(X) = (0-3)^2 \times 1/3 + (2-3)^2 \times 1/3 + (7-3)^2 \times 1/3 = 8.66$. Therefore by the Central Limit Theorem, $\bar{X}$ is approximately $N(3, 8.66/100)$. We should reject the null hypothesis if $\bar{X}$ is large since the mean of the distribution in the alternative hypothesis is larger than that in the null hypothesis. Denote the rejection region by $\bar{X} > c$. Then

$$\begin{aligned}
P(\bar{x} > c|H_0) &= 0.01 \\
P(Z > \frac{c-3}{\sqrt{0.0866}}) &= 0.01 \\
\frac{c-3}{0.294} &= 2.33 \\
c &= 3.685
\end{aligned}$$

Grade Distribution