Stat/For/Hort 571
Midterm II, Fall 2003
Brief Solutions

1. (a) $s_p = 44.52$, $t_{0.005,33} = 2.73$ from Table B. 99% CI is $\bar{y}_A - \bar{y}_B \pm t_{0.005,n_A+n_B-2} s_p \sqrt{1/n_A + 1/n_B} = 240 - 190 \pm 2.73 * 44.52\sqrt{1/15 + 1/20} = (8.44, 91.56)$.

   (b) CI does not cover 0, so reject H_0 at 1% level. Alternatively, t score is 3.29 and 2-sided p-value is 0.0024.

2. (a) First verify that sample size is large. That is, $\hat{p} = 32/60 = 0.53$ and $n_1\hat{p}, n_1(1-\hat{p}), n_2\hat{p}, n_2(1-\hat{p})$ all ≥ 14. Thus can use normal approximation $Z = (\hat{p}_1 - \hat{p}_2)/(s_p\sqrt{1/n_1 + 1/n_2}) \approx N(0,1)$ under $H_0 : p_1 = p_2$. Here $s_p = \sqrt{\hat{p}(1-\hat{p})} = 0.499$ and z-score is $z = (0.67 - 0.40)/(0.499\sqrt{2/30}) = 2.07$. One-sided p-value from Table A is 0.019. Argue for one-sided or two-sided test. Conclude there is strong evidence for significant symptom reduction.

   (b) Binomial assumptions: 2 outcomes, independent trials, equal probability (may be different for trt 1 and trt 2). Comment on 2 independent samples and normal approximation.

3. ANOVA table: Verify SSAgent $= 231.58 = 4388 - 1288^2/38 = 10(34 - 33.89)^2 + 10(30 - 33.89)^2 + 18(36 - 33.89)^2$. Show SSError $= 1062 = 9(5^2) + 9(5^2) + 17(6^2) =$ SSTot - SSAgent; SSTot $= 44950 - 1288^2/38$.

Source	df	SS	MS
Agent	2	231.58	115.79
Error	35	1062.00	30.343
Total	37	1293.58	

   F-value = 3.861, p-value = 0.032. Reject H_0 : all means identical at 5% level.

4. (a) We want $2 * z_{0.05} * \sigma/\sqrt{n} \leq 10$ where $z_{0.05} = 1.645$, $\sigma = 10$. Thus $n \geq (2 * z_{0.05} * \sigma/10)^2 = (2*1.645*10/10)^2 = 10.82$. Round up to 11.

   (b) CI always increases with confidence level, so need larger sample size to keep width at 10 units. Or repeat computation as above to get $n \geq 15.37$, with $z_{0.025} = 1.96$, round up to 16.

   (c) NOTE: Typo in original exam. Should have $H_0 : \mu = 200$ and $H_A : \mu = 210$ for 10 fields (not pairs). Find critical value based on power $= P(\bar{Y} \geq c|\mu = 210) = P\left(Z \geq \frac{c-210}{\sigma/\sqrt{n}}\right) = 0.90$. $c = 210 - 1.28 * 10/\sqrt{10} = 205.95$. Now find $\alpha = 2P(\bar{Y} \geq c|\mu = 200) = 2P\left(Z \geq \frac{c-200}{\sigma/\sqrt{n}}\right) = 2P(Z \geq 1.88) = 0.060$.

5. (a) $E(X_1) = \mu = \sum x p_x = 2.5$, $Var(X_1) = \sigma^2 = \sum(x-\mu)^2 p_x = 0.917$. Let $X = \sum X_i$ be the sum of n draws. $X \approx N(n\mu, n\sigma^2)$ by CLT. Here $n = 20$: $X \approx N(50, 18.33)$.

   (b) Two-sided test–could be either extreme. $H_0 : \mu = 50$, $H_A : \mu \neq 50$. Observed $x = 10 * 1 + 10 * 2 = 30$. z-score is $z = (x - 50)/\sqrt{18.33} = -4.67$. $2P(Z \geq 4.67) < 0.0001$. Assume random sample of 20 draws with replacement from opaque bag. That is, draws are independent. Also using normal approximation due to large n.

Grade Distribution

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90-99:22
80-89:55
70-79:39   mean = 80, median = 81
60-69:9
50-59:3
20-49:3
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