Assignment 10 — Due December 5, 2003

1. The following is a subset from a larger experiment described below. The dry sheer strength (in pounds per square inch) of birch plywood bonded with 4 different resin glues (A, B, C, and D) were compared. Three pieces of plywood were tested with each glue type.

A	504	499	473
B	455	483	476
C	500	502	483
D	520	511	562

   Compute an ANOVA table for these data (using a hand calculator), including all relevant sums of squares, mean squares, and degrees of freedom.

2. The following data represent the complete set of measurements of the dry sheer strength (in pounds per square inch) of birch plywood, bonded with 5 different resin glues A, B, C, D, and E. Eight pieces ($n = 8$) of plywood were tested with each glue type. (Note: You should use R for most of the computations on parts (a) and (b). See comments below.)

A	504	499	473	458	479	468	463	517
B	455	483	476	493	498	466	492	484
C	500	502	483	519	538	528	509	500
D	520	511	562	533	495	543	513	540
E	514	496	503	482	512	532	510	499

   (a) Make 5 side-by-side stem-leaf displays.

   Remark: A brief visual inspection of the data should be a routine part of a one-way ANOVA. An examination of the side-by-side stem-leaf displays can help give a feeling for differences among glue types, as well as aid in checking for approximate normality and for checking that the variances are roughly similar.

   (b) Compute an ANOVA table for these data, including all relevant sums of squares, mean squares, degrees of freedom. Also compute the relevant F.

   (c) State the statistical model underlying the procedures in the analysis of variance as applied to these data. Define symbols used and make clear all distributional assumptions. (You need not formally verify the assumptions.)

   (d) State in words and symbols the null hypothesis and alternative hypothesis appropriate to this problem which is tested by the usual F-test. Find the p-value for the test.

   (e) Assume glue A is a "standard" glue and B, C, D, and E are new experimental glues. Find a 99% CI for $\mu_A - \frac{1}{4}(\mu_B + \mu_C + \mu_D + \mu_E)$. Interpret your CI.

   (f) Conduct a test for the following:

$$H_0 : \frac{1}{2}(\mu_B + \mu_D) = \frac{1}{2}(\mu_C + \mu_E) \quad \text{vs} \quad H_A : \frac{1}{2}(\mu_B + \mu_D) \neq \frac{1}{2}(\mu_C + \mu_E)$$

   Hints for using R are in Appendix 11.9. You may use the data file
   `http://www.stat.wisc.edu/~st571-1/data/sheer.dat`

3. To study the effect of relative humidity (RH) on the weight of newly formed pupae, environments of 70% RH, 20% RH, and 1% RH were used. A number of first-stage larvae were placed on the same diet at each of these RH levels with the following weights (in mg) recorded for the newly formed pupae.

   70% RH: 18.5, 21.0, 23.5, 26.2, 22.5, 24.0, 21.5, 24.8, 20.5, 24.5, 19.5, 22.5, 21.5

   20% RH: 19.0, 18.0, 21.3, 19.5, 18.3, 18.8

1% RH: 14.7, 16.1, 20.2, 16.7, 18.1, 14.7, 18.3, 17.8, 17.2, 15.8, 17.5

(a) Compute a 1-way ANOVA table including degrees of freedom, sums of squares and mean squares. Again use R for the ANOVA computations. Enter the data from the 70% RH group into column `y1`; enter the 20% RH data into `y2`; and enter the 1% data into `y3`. Then, use the `lm` and `anova` command to do the analysis of variance. (See also "An Example of ANOVA using R" under Appendix 11.9. Replace `n=rep(7,3)` with `n=c(13,6,11)`)

(b) Is there evidence that RH affects the average weight of newly formed pupae?

(c) Compute 95% confidence limits for the difference in mean weights between pupae at 20% RH and pupae at 70% RH.

(d) Perform a test for the homogeneity of variances.

(e) Compute 99% confidence limits for the underlying error variance.

4. The weight of corn is known to depend on the amount of nitrogen available for plant growth. A nitrogen (N) supplement is developed to enhance plant growth. Five corn plants are grown with no N supplement; five plants are grown with 1 unit of N supplement: and similarly five plants are grown with 2,3,4 and 5 units of N supplement. The recorded weights (in arbitrary units) of the ears of corn at the end of the growing season are given below.

Group 1 0 N	Group 2 1 N	Group 3 2 N	Group 4 3 N	Group 5 4 N	Group 6 5 N
12.4	14.5	14.2	22.1	26.1	22.8
15.7	11.4	18.0	17.4	19.7	27.1
10.1	18.9	20.5	20.6	23.1	26.2
11.7	16.3	15.1	19.0	21.0	19.8
12.8	13.8	18.5	19.1	24.2	25.3

(a) Perform a usual ANOVA analysis with an ANOVA table and F-test. (Again, use R. It is not necessary to check the assumptions for this question.) Interpret the results of your F-test.

(b) Let X be the number of units of N supplement and let Y be the weight of an individual ear of corn. Plot Y versus X. Comment on the plot.

(c) Test the following hypothesis where μ_i is the mean weight of an ear of corn from Group i : $H_0 : -5\mu_1 - 3\mu_2 - 1\mu_3 + 1\mu_4 + 3\mu_5 + 5\mu_6 = 0$. H_A is the two-sided alternative.

Note: The given contrast is useful in determining if the relationship between weight and units of N follows a linear trend. Try to convince yourself that this is so.

5. Below are the lengths (in mm) of 10 cocoons from 7 particular races of silkworm.

Race A: 29, 30, 30, 31, 31, 31, 32, 33, 34, 36

Race B: 27, 29, 30, 31, 31, 32, 32, 33, 33, 34

Race C: 26, 27, 28, 28, 29, 30, 30, 30, 31, 33

Race D: 30, 32, 33, 33, 33, 34, 35, 36, 36, 37

Race E: 31, 33, 33, 34, 34, 34, 35, 36, 36, 37

Race F: 26, 27, 28, 28, 28, 29, 30, 30, 31, 33

Race G: 29, 31, 32, 32, 34, 34, 35, 35, 36, 38

(a) Compute a 1-way ANOVA table. (Use R — the data are stored in the file `cocoon.dat` . Carry out the customary F-test. State explicitly what is being tested defining all symbols used.

(b) State the assumptions you are making (again defining all symbols used). Where possible, verify the assumptions. (You do not have to verify formally the homogeneity of variance assumption.)

(c) Using the LSD procedure and the Q method (at the 0.05 level), find which pairs of means are and are not significantly different from one another. (Use the form of display discussed in class.) Summarize your results, comparing the 2 procedures.

(d) Assuming race A is a "standard" race, use the Bonferroni procedure to test for equality of the mean of A with the mean of each of the other races (B, C, D, E, F, G). Use an experiment-wise error rate of 0.05.

Readings:

- Weeks 13–14: Course Notes, Chapter 12.