Assignment 7 — Due October 31, 2003

1. The purpose of this problem is to illustrate by simulation the "long-run" performance of confidence intervals. Recall that when constructing 95% confidence intervals for μ, in the long run, 95% of all confidence intervals constructed will contain the true value of μ.

   (a) In this problem you will simulate 200 samples of size 15 from $N(62, 9^2)$ as in Problem 1 of Assignment # 6. Use R to generate 95% confidence intervals for each sample. Of the 200 intervals you compute, how many contain the value $\mu = 62$? [Appendix 7.6.2 has some R code to conduct such simulations.]

   (b) In part (b) of Problem 1 on Assignment # 6 you determined how many times $H_0 : \mu = 62$ was rejected at $\alpha = 0.05$. How do your results above from (a) on this confidence interval simulation compare with your $\alpha = 0.05$ results from the hypothesis testing simulation? Discuss the relationship.

2. The purpose of this exercise is to indicate how widely the appearance of data sets comprised of small numbers of observations from normal distributions can vary.

   (a) Recall the `rnorm` command used on Assignment # 6 Problem 1. Use this command to generate 3 independent data sets, each of size 10 ($n = 10$), from a N(75,100) distribution. For each of these data sets construct a stem-leaf display and a normal scores plot. Normal scores plots can be produced using the `qqnorm` command. The data in a normal scores plot should fall roughly on a straight line if the distribution being sampled is a normal distribution. [See the R command `qqnorm` described in Appendix 8.5.]

   (b) Repeat part (a), except this time generate 3 independent data sets, each of size 40 from a N(75, 100) distribution.

   (c) Write a concise paragraph summarizing your findings.

3. Given below are 12 (randomly selected) observations of bacteria counts on soybean leaves — your ultimate goal is to construct a confidence interval based on these data. It is felt a priori that either original units or log units are appropriate for the data. Use normal scores plots to determine which units are best suited for inference using normal theory. Comment explicitly on the appearance of the normal scores plots. Based on the units that you choose (original or log) construct a 95% confidence interval for the mean. (Note — You may use either base e or base 10 for your logs.)

4.15×10^6	1.75×10^7	1.35×10^6	3.32×10^6	3.28×10^7	4.48×10^6
3.22×10^6	7.94×10^6	4.80×10^6	2.29×10^6	1.67×10^6	3.56×10^6

4. An animal nutritionist has designed a diet for laboratory rabbits that is intended to cause 3 week old rabbits (from a standard breed) to have a mean weight of $\mu = 120$ grams. From previous experience, it is known that rabbit weights are approximately normally distributed and that the population standard deviation of weight at 3 weeks is $\sigma = 12.0$ grams. To explore the question as to whether or not the diet achieves the desired result (mean weight 120 grams), a researcher will feed 24 randomly selected rabbits the diet according to the recommended schedule.

The researcher will reject the hypothesis that the diet achieves the desired result if $\bar{X} < 113.2$ or $\bar{X} > 126.8$. Find the power of this test for the following values of μ : 108, 112, 116, 120, 124, 128, 132. Draw the power curve. What is meant by the power at $\mu = 120$?

5. An entomologist claims that the mean wing length of a particular species of insect is at most 15.0 mm. An $\alpha = 0.025$ test is desired to test the claim. The researcher feels that if the true mean wing length is 17.2 mm, it would be important to have a 90% chance of rejecting the claim. Assume that the wing lengths are normally distributed and that the standard deviation of wing lengths is 3.8 mm ($\sigma = 3.8$).

   For parts (a), (b), and (c) of this problem, you are expected to derive the result in a manner similar to that used in Section 9.3. You may use Equations 9.8 or 9.12 (as appropriate) only to check your answer, not to derive it.

   (a) Determine the sample size for this test. Find the rejection criterion.

   (b) Repeat (a) for an $\alpha = 0.10$ test.

   (c) Repeat (a) with $\alpha = 0.025$ but with a power of 0.75 at μ =17.2.

   (d) Without performing any additional computations, explain how α, power (at a particular alternative), and sample size are related.

Readings:

- Week 8: Course Notes Chapter 8, Chapter 9