Stat/For/Hort 571 — Fall 2003

Assignment 4 — Brief Solutions

2 (a) For I, $Var(\bar{X}) = \frac{\sigma^2}{n} = \frac{5^2}{2} = 12.5$; For II, $Var(\bar{X}) = \frac{\sigma^2}{n} = \frac{8^2}{5} = 12.8$. So, I is more precise.

(b) $\bar{X} \sim N(63, 8^2/5)$.So,

$$
\begin{aligned}
P(60 \le \bar{X} \le 65) &= P(\frac{60-63}{\sqrt{8^2/5}} \le Z \le \frac{65-63}{\sqrt{8^2/5}}) \\
&= P(-0.8385 \le Z \le 0.559) \\
&= 1 - P(Z > 0.8385) - P(Z > 0.559) \\
&= 1 - 0.2005 - 0.2871 = 0.5118
\end{aligned}
$$

(c) $Var(\frac{X+Y}{2}) = \frac{Var(X)+Var(Y)}{4} = 22.25$
So, the standard deviation is 4.717

3 $P(\sum_{i=1}^{20} X_i < 40) = P(Z < \frac{40-2.2*20}{\sqrt{20*0.2}}) = P(Z < -2) = 0.02275$

4 (a)

$$X \sim B(n = 180, p = 0.3)$$

By normal approximation, $X_{na} \sim N(np, np(1-p)) = N(54, 37.8)$. So,

$$
\begin{aligned}
P(X \ge 50) &\approx P(X_{na} \ge 50) \\
&= P(\frac{X_{na} - 54}{\sqrt{37.8}} \ge \frac{50-54}{\sqrt{37.8}}) \\
&= P(Z \ge -0.6506) \\
&= 0.7423
\end{aligned}
$$

(b) By normal approximation, $Y_{na} \sim N(p, p(1-p)/n) = N(0.3, 0.001167)$. So,

$$
\begin{aligned}
P(0.25 < Y < 0.40) &\approx P(0.25 < Y_{na} < 0.40) \\
&= P(-1.46385 < Z < 2.9277) \\
&= 0.9267
\end{aligned}
$$

5 (a) $a = 77.9167$, $b = 1.8333$

(b) $a = 44.3971$, $b = 11.1176$

(c) $0.950 \le P(S^2 \le 46) \le 0.975$, $0.005 \le P(S^2 \ge 58) \le 0.01$

(d) $\sigma^2 = 6.9314$, $b = 3.1365$

(e) μ

(f) $a = 9.737$, $b = 4.515$

(g) 0.4216, 0.8860

6 Let

$$X_1 = \begin{cases} 1 & : \text{ if Ray shows up at 4pm} \\ 0 & : \text{ otherwise} \end{cases}$$

$$X_2 = \begin{cases} 1 & : \text{ if Felix shows up at 4pm} \\ 0 & : \text{ otherwise} \end{cases}$$

Then, $X_1 \sim B(1, 0.8)$, $X_2 \sim B(1, 0.6)$, and X_1 and X_2 are independent.

Let $X = X_1 + X_2$.

(a)

$$\begin{aligned} P(X \leq 1) &= P(X = 0) + P(X = 1) \\ &= P(X_1 = 0, X_2 = 0) + P(X_1 = 1, X_2 = 0) + P(X_1 = 0, X_2 = 1) \\ &= P(X_1 = 0)P(X_2 = 0) + P(X_1 = 1)P(X_2 = 0) + P(X_1 = 0)P(X_2 = 1) \\ &= (0.2)(0.4) + (0.8)(0.4) + (0.2)(0.6) \\ &= 0.52 \end{aligned}$$

(b) They are not mutually exclusive.

(c)

x	0	1	2
$p(x)$	0.08	0.44	0.48

(d)

$$\begin{aligned} E(X) &= E(X_1 + X_2) \\ &= E(X_1) + E(X_2) \\ &= 0.8 + 0.6 \\ &= 1.4 \end{aligned}$$