Solutions for Homework 5

1. Normal data with $\sigma = 8$. Test $H_0: \ \mu = 60$ versus $H_A: \ \mu \neq 60$.

   (a) $n = 16$ and $\bar{x} = 63$,
   $z = \frac{\bar{x}-60}{\sigma/\sqrt{n}} = \frac{63-60}{8/\sqrt{16}} = 1.5$,
   $p - value = P(Z \leq -1.5) + P(Z \geq 1.5) = 2P(Z \geq 1.5)$
   $= 2(.0668) = .1336 > 0.05$
   Not significant at 5%.

   (b) $n = 64$ and $\bar{x} = 63$,
   $z = \frac{\bar{x}-60}{\sigma/\sqrt{n}} = \frac{63-60}{8/\sqrt{64}} = 3$,
   $p - value = P(Z \leq -3) + P(Z \geq 3) = 2P(Z \geq 3)$
   $= 2(.0013) = .0026 < 0.05$
   Significant at 5%.

   (c) $\sigma = 4$, $n = 16$ and $\bar{x} = 63$,
   $z = \frac{\bar{x}-60}{\sigma/\sqrt{n}} = \frac{63-60}{4/\sqrt{16}} = 3$,
   $p - value = P(Z \leq -3) + P(Z \geq 3) = 2P(Z \geq 3)$
   $= 2(.0013) = .0026 < 0.05$
   Significant at 5%.

   (d) Conclusion: when σ gets smaller or the sample size gets larger, the p-value will get smaller for the same test.

2. (a) stem-leaf display:

```
10|7
13|236
14|5
15|2
16|5
17|6
18|7
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   (b) n=9, $\sigma = 220$. Test $H_0: \ \mu = 1620$ versus $H_A: \ \mu \neq 1620$,
   $\bar{x} =$
   $z = \frac{\bar{x}-1620}{\sigma/\sqrt{n}} = \frac{1480-1620}{\sqrt{48400/9}} = -1.91$,
   $p - value = P(Z \leq -1.91) + P(Z \geq 1.91) = 2P(Z \geq 1.91) = 2(0.0281) = 0.0562 > 0.05$.
   Not significant at $\alpha = 0.05$, $\alpha = 0.01$ Significant at $\alpha = 0.1$. We accept the claim at leve 5% and 1%, reject it at level 10%.

3. (a) stem-leaf display:

```
-4| 3
-3| 86
-3| 31
-2| 977
-2| 33210
-1| 96
-1| 3
-0| 9
-0| 2
```

(b) Test $H_0: \ \mu = -3.0$ versus $H_A: \ \mu \neq -3.0$,
Assume normal data, $\bar{x} = -2.4$, $s = 1.036$, $n = 18$.
$t = \frac{\bar{x}-(-3.0)}{s/\sqrt{n}} = \frac{-2.4-(-3.0)}{1.036/\sqrt{18}} = 2.457$,
$p - value = P(T \leq -2.457) + P(T \geq 2.457) = 2P(T \geq 2.457)$
$0.01 < P(T \geq 2.457) < 0.025$ (d.f=17), so p-value is between 0.02 and 0.05.
The claim will be rejected at $\alpha = 0.05$. There's a moderate evidence against the null hypothesis.

4. $m = 80$, $X \sim Bin(80, p)$. Test $H_0: \ p = 0.70$ versus $H_A: \ p \neq 0.70$.

   (a) $p - value = 2P(X \leq 50) = 2P(Z_{NA} \leq \frac{50-80(.70)}{\sqrt{80(.70)(.30)}})$
   $= 2P(Z_{NA} \leq -1.46) = 2(.0721) = .1442$
   No evidence against the claim.

   (b) $n = 320$, $x = 200$, $X \sim Bin(30, p)$,
   $p - value = 2P(X \leq 200) = 2P(Z_{NA} \leq \frac{200-320(.70)}{\sqrt{320(.70)(.30)}})$
   $= 2P(Z_{NA} \leq -2.92) = 2(.0018) = .0036$
   There's strong evidence against the claim.

   (c) With the same proportion $\frac{x}{n}$, the larger n, the smaller p=value we'll obtain.

5. Let X = number of contaminated wells, $X \sim Bin(10, p)$. Test $H_0 : p = 0.45$ versus $H_A : p \neq 0.45$.
   $p - value = 2(P(X = 8|H_0) + P(X = 9|H_0) + P(X = 10|H_0))$
   We cannot use the normal approximation, because $np = 10(0.45) = 4.5 < 5$
   $P - value = 0.0548 > 0.05$.
   Or you can do onesided test, get P-value=0.0274, we decline null hypothesis

6. (a) False. It depends on the standard deviation.
   (b) True