STAT 571, Solution for Assignment #6

October 22, 2003

1. (a). $\alpha = 0.1$, on average 200 (0.1) = 20 times that the H_o will be rejected;
   $\alpha = 0.05$, on average 200 (0.05) = 10 times that the H_o will be rejected;
   $\alpha = 0.01$, on average 200 (0.01) = 2 times that the H_o will be rejected.

   (b). Perform the simulations. Compare the p-values (calculated by R) with each level of α, and count the times that H_o is actually rejected. We will find these realized results are close to the on average results from part (a).

2. $H_o : \sigma^2 = 40000, \quad H_A : \sigma^2 \neq 40000.$
   $V^2 = \frac{(n-1)S^2}{\sigma^2}|_{H_o} = \frac{8S^2}{0.01},$
   $v^2 = \frac{8(61991)}{40000} = 12.40.$
   p-value $= 2P(V^2 \geq 12.40)$.
   By looking up the table, we find $0.2 <$ p-value < 0.5.

3. $n = 9, \quad \bar{X} = 2.5156$
   $s = 0.276$.

   (a). $1-\alpha$ C. I. for μ is $\bar{x} - T_{n-1,\frac{\alpha}{2}} \frac{s}{\sqrt{n}} \leq \mu \leq \bar{x} + T_{n-1,\frac{\alpha}{2}} \frac{s}{\sqrt{n}}$.
   90 % C. I. for μ: $2.344 \leq \mu \leq 2.687$,
   95 % C. I. for μ: $2.303 \leq \mu \leq 2.728$.

   (b). $1-\alpha$ C. I. for σ^2: $\frac{(n-1)s^2}{\chi^2_{\frac{\alpha}{2}}} \leq \sigma^2 \leq \frac{(n-1)s^2}{\chi^2_{1-\frac{\alpha}{2}}}$.
   90 % C. I. for σ^2: $0.0393 \leq \sigma^2 \leq 0.2230$,

95 % C. I. for σ^2: $0.0348 \le \sigma^2 \le 0.2796$.

(c). 95 % C. I. for μ: $2.422 \le \mu \le 2.609$,
The C.I is narrower than that in part (a).

4. (a). 90 % C. I. for p:

$$\hat{p} - Z_{\frac{\alpha}{2}}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \le p \le \hat{p} + Z_{\frac{\alpha}{2}}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

$$0.625 - 1.645\sqrt{\frac{625(.375)}{80}} \le p \le 0.625 + 1.645\sqrt{\frac{625(.375)}{80}}$$

$$0.536 \le p \le 0.714.$$

(b). 90 % C. I. for p:

$$\hat{p} - Z_{\frac{\alpha}{2}}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \le p \le \hat{p} + Z_{\frac{\alpha}{2}}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

$$0.625 - 1.645\sqrt{\frac{625(.375)}{320}} \le p \le 0.625 + 1.645\sqrt{\frac{625(.375)}{320}}$$

$$0.5805 \le p \le 0.6695,$$

The C. I is narrower than that in part (a).

5. (a). Population $N(\mu, 16)$, so 90 % C. I. for μ:

$$\bar{x} - Z_{0.05}\frac{\sigma}{\sqrt{n}} \le \mu \le \bar{x} + Z_{0.05}\frac{\sigma}{\sqrt{n}}.$$

So $W(length) = (\bar{x} + Z_{0.05}\frac{\sigma}{\sqrt{n}}) - (\bar{x} - Z_{0.05}\frac{\sigma}{\sqrt{n}}) = 2Z_{0.05}\frac{\sigma}{\sqrt{n}} = 0.5$.
That is $2(1.645)\frac{4}{\sqrt{n}} = 0.5$.
We get $n = \left(\frac{2(1.645)(4)}{0.5}\right)^2 = 692.74 \approx 693$.

(b). 90 % C. I. for μ: $19.51 \le \mu \le 21.88$, $\Longrightarrow$ we reject H_o at $\alpha = 10\%$;
95 % C. I. for μ: $19.24 \le \mu \le 22.16$, $\Longrightarrow$ we accept H_o at $\alpha = 5\%$;
99 % C. I. for μ: $18.60 \le \mu \le 22.79$, $\Longrightarrow$ we accept H_o at $\alpha = 1\%$.

(c). i. $T = \frac{Z-\mu_0}{s/\sqrt{n}} = \frac{20.7-22}{2.042/3.162} = -2.013.$

p-value $= 2P(T \leq -2.013) = 0.07$ (from R).

At $\alpha = 10\%$, reject H_o;

At $\alpha = 5\%$ and 1% , accept H_o.

This is consistent to results from part (b).

ii. Find 99.5 % C. I. for μ.

df $= 9$, $\alpha = 0.005$, then $\alpha/2 = 0.0025$.

$P(T \leq t^*) = 0.0025$, then $t^* = -3.6897$, (from R).

So 99.5 % C. I. for μ is:

$$20.7 - 3.6897\left(\frac{2.04}{3.16}\right) \leq \mu \leq 20.7 + 3.6897\left(\frac{2.04}{3.16}\right)$$

$$18.32 \leq \mu \leq 23.08.$$