Solutions for Homework 8

1. (a) Null hypotheses H_0: $p = 0.25$, alternative hypotheses H_A: $p > 0.25$. p is the chance of remission with the treatment.

   (b) α = P(Type I error) =P(reject $H_0|H_0$) $= P(X \geq 150|p = 0.25)$

   Using normal approximation to Binomial data,

   $\alpha = P(Z > \frac{150-500(.25)}{\sqrt{500(.25)(.75)}}) = P(Z > 2.582) = .0049$

   (c) Power = P(reject $H_0|p = 0.35) = P(X \geq 150|p = 0.35) = P(Z > \frac{150-500(.35)}{\sqrt{500(.35)(.65)}})$

   $= P(Z > -2.344) = .9905$

   (d) P-value $= P(X \geq 154|p = 0.25) = P(Z > \frac{154-500(.25)}{\sqrt{500(.25)(.75)}}) = P(Z > 2.995) = .0014$

2. (paired experiment)

   The yield differences of the two varieties are: -2.2 -1.3 0.0 -2.0 -1.8 -0.8 1.2 -4.1 -1.6 -0.9

   (a) We need to assume that the yield differences of the two varieties is normally distributed. The normality assumption can be checked by a normal score plot:

```
MTB > nscores c1 c2
MTB > plot c1 c2

          -
 C1       -                                                      *
          -
          -
       0.0+                                             *
          -
          -                                  *    *
          -                              *
          -                      *   *
      -2.0+           *     *
          -
          -
          -
          -
      -4.0+  *
          -
          -
           --------+---------+---------+---------+---------+--------C2
               -1.20     -0.60      0.00      0.60      1.20
```

   (b) Test $H_0 : \mu_1 - \mu_2 = 0$ versus $H_A : \mu_1 - \mu_2 \neq 0$.

   $\bar{d} = -1.35$, $s_d = 1.407$, the test statistic $t = \frac{-1.35-0}{1.407/\sqrt{10}} = -3.034$,

   using a t-distribution with degree of freedom 9, we can get the p-value of the test:

   $2(.005) <$ p-value $= 2P(T < -3.034) < 2(.01)$, i.e., $.01 <$ p-value $< .02$.

   (c) A 99% C.I. for $\mu_1 - \mu_2$ is given by:

$$\bar{d} \pm t_{.005} \frac{s_d}{\sqrt{10}} = -1.35 \pm 3.250 \frac{1.407}{\sqrt{10}} = -1.35 \pm 1.446 = (-2.796, 0.096).$$

   The interval covers 0, so H_0 won't be rejected at $\alpha = 0.01$.

3. (independent samples with same sample size)

   Species A: $\bar{x}_1 = 4.764$, $s_1^2 = 0.25$, $n_1 = 9$,

   Species B: $\bar{x}_2 = 4.242$, $s_2^2 = 0.29$, $n_2 = 9$.

   (a) We assume independence, normality and equal variance.

(b) Test $H_0 : \mu_1 - \mu_2 = 0$ versus $H_A : \mu_1 - \mu_2 \neq 0$.
since $n_1 = n_2$, $s_p^2 = \frac{s_1^2 + s_2^2}{2} = 0.27$, $s_{(\bar{X}_1 - \bar{X}_2)} = \sqrt{2s_p^2/9} = 0.245$.
Test statistic $t = \frac{\bar{X}_1 - \bar{X}_2}{s_{(\bar{X}_1 - \bar{X}_2)}} = \frac{4.764 - 4.242}{0.245} = 2.13$.
$.02 <$ p-value $= 2P(T > 2.13) \; < .05$ (degrees of freedom of the t-distribution is 16). We have moderate evidence that the two Species don't have the same mean egg weight.

(c) A 95% C.I. for $\mu_1 - \mu_2$ is given by:

$$(x_1 - x_2) \pm t_{.025} s_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}} = (4.764 - 4.242) \pm 2.120\sqrt{.27}\sqrt{\frac{1}{9} + \frac{1}{9}}$$

$$= .522 \pm .5197 = (.0023, 1.0417)$$

(d) Test $H_0 : \mu_1 - \mu_2 = -0.5$ versus $H_A : \mu_1 - \mu_2 \neq -0.5$.
Test statistic $t = \frac{(\bar{X}_1 - \bar{X}_2) - (-0.5)}{s_{(\bar{X}_1 - \bar{X}_2)}} = \frac{(4.764 - 4.242) + 0.5}{0.245} = 4.17$.
p-value $= 2P(T > 4.17) < .002$. We have strong evidence that the mean egg weight of Species B eggs doesn't equal the mean weight of Species A eggs plus 0.5.

4. (Independent samples with unequal sample sizes)

Tree 1: $\bar{x}_1 = 145.5$, $s_1^2 = 164.76$, $n_1 = 13$, Tree 2: $\bar{x}_2 = 154.1$, $s_2^2 = 162.77$, $n_2 = 10$,

Assuptions: independent random samples from normal populations with equal variances. We can use Levene's method to check the assumption of equal variances($\sigma_1^2 = \sigma_2^2$):

1) The median for Tree 1 is 145, and the median for Tree 2 is 155.5.

2) Calculate the absolute value of all deviations from the median. We get

Tree 1: 24 25 12 0 2 12 5 9 17 6 2 4 6

Tree 2: 2.5 8.5 18.5 26.5 13.5 12.5 6.5 7.5 3.5 23.5

3) Delete the value 0 in Tree 1.

Tree 1: 24 25 12 2 12 5 9 17 6 2 4 6

Tree 2: 1.5 8.5 14.5 15.5 29.5 1.5 17.5 1.5 7.5 12.5

4) Perform a T-test for comparing the means of the two lists of numbers(with variances assumed equal). There's no evidence that the variances σ_1^2 and σ_2^2 differ, by calculating P-value.

So, we will perform a T-test assuming equal variances.

Test $H_0 : \mu_1 - \mu_2 = 0$ versus $H_A : \mu_1 - \mu_2 \neq 0$.
$s_p^2 = \frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2} = \frac{(13-1)164.76 + (10-1)162.77}{13+10-2} = \frac{3442.05}{21} = 163.91$,
$s_{(\bar{X}_1 - \bar{X}_2)} = \sqrt{s_p^2(\frac{1}{n_1} + \frac{1}{n_2})} = 5.38$.
Test statistic $t = \frac{\bar{X}_1 - \bar{X}_2}{s_{(\bar{X}_1 - \bar{X}_2)}} = \frac{145.5 - 154.1}{5.38} = -1.598$.
p-value $= 2P(T > 1.59) \; < .20$ (degrees of freedom of the t-distribution is 21). We have stropng evidence that the mean weights of the samaras of tree 1 and tree 2 differ.

5. Test $H_0 : \sigma^2 = 4$ versus $H_A : \sigma^2 > 4$ at $\alpha = 0.05$. $n = 15$.

We will reject H_0 if the sample variance $s^2 > a$,

$0.05 = \alpha = P(\text{reject } H_0|H_0) = P(s^2 > a|\sigma^2 = 4) = P(V^2 = \frac{(15-1)s^2}{4} > \frac{(15-1)a}{4})$, where V^2 has χ^2-distribution with $d.f. = 14$.

So, $\frac{(15-1)a}{4} = 23.68$, $a = \frac{4(23.68)}{15-1} = 6.77$. Rejection region: we will reject H_0 if $s^2 > 6.77$.

Power at $\sigma^2 = 13$ is

$P(\text{reject } H_0|\sigma^2 = 13) = P(s^2 > 6.77|\sigma^2 = 13) = P(V^2 > \frac{(15-1)6.77}{13}) = P(V^2 > 7.29) \approx .925.$