Syllabus for STAT 709 Mathematical Statistics

Last update: Sep 15, 2024

Basic course information

Institution: Department of Statistics, University of Wisconsin—Madison

Course number: STAT 709

Course title: Mathematical Statistics

Number of credits: 4

Instruction mode: in-person

Learning activities to fulfill credits: Lectures twice a week (75 mins), one discussion session each week. Assigned reading and weekly homework.

Lecture info

- **Instructor:** Yiqiao (Joe) Zhong, (yiqiao.zhong@wisc.edu)
- **TA:** Jingyang Lyu (JLYU55@wisc.edu)
- **Office hour**: Mondays 10 am — 11 am (Joe), Thursday 3:00 pm — 4:00 pm at SMI 133 (Jingyang)
- **Time**: Mondays, Wednesdays 11:00 am — 12:15 pm
- **Location**: VAN VLECK B223

Course description

STAT 709 is one of the core courses in the curriculum of the Statistics Ph.D. program. It is the first course in the mathematical statistics series and followed by STAT 710. This course introduces the fundamentals of statistical theory and prepares Ph.D. students for statistical research.

Prerequisites

Graduate/professional standing.

Learning outcomes

At the end of the course, students will be able to:

- Define statistical estimation problems using rigorous mathematical languages
- Derive basic statistical properties of an estimator such as bias and variance
- Analyze and compare different statistical estimators, and articulate the strengths and weaknesses of each method
- Apply theoretical tools such as concentration inequalities to new statistical problems in the form of mathematical proofs
- Articulate the distinctions between classical large-sample statistical theory and high-dimensional statistical theory, and critique recent statistical research

Course policy

Grading

Breakdown: Assignment 25%, midterm 25%, final 50%, 100 points
Bonus points
- Homework (up to 5 points)
- Piazza participation (up to 2 points)
- Scribe (up to 2 points)

Grade distribution

A: (80, 100]; AB: (70, 80]; B: (60, 70]; BC: (50, 60]; C: (40, 50]; D: (30, 40]; F: [0, 30].

Assignment

- **Exercises:** most exercises refer to Jun Shao's Mathematical Statistics (2nd edition)
- **Submission format:** either latex or handwriting is fine; must submit via Canvas
- **Frequency:** normally posted by Sunday evening every week 😢
- **Due:** roughly one week after HW is posted
- **Late submission:** A deduction will be applied before solutions are posted, zero grade after solutions are posted 😱

Exams

- **Midterm**: in-class, date TBD (likely at the end of October), same location as lectures (75 mins), 2 pages of cheatsheet (double-sided, A4 size), no need for calculators
- **Final exam**: A fixed date in December, location TBD (2 hrs), 2 double-sided pages of cheatsheet (double-sided, A4 size), no need for calculators

Communication

- Office hour: 1 hour by instructor, and 1 hour by TA each week
- Ask course-related questions on Canvas "Discussions"; may choose to be anonymous

- Use emails for emergent situations such as sickness

Course schedule and topics

Many topics will be based on lecture notes.

The official textbook and references are the following.

Textbook: Mathematical Statistics: Basic Ideas and Selected Topics, Volume I, Second Edition, by Bickel and Doksum

References:

- Jun Shao, Mathematical Statistics: Exercises and Solutions, Springer 2005
- Roman Vershynin, High-dimensional Probability: An Introduction with Applications in Data Science, Cambridge 2018
- Martin Wainwright, High-dimensional Statistics: A Non-Asymptotic Viewpoint, Cambridge 2019

Part 1: Mathematical Foundations of Statistical Theory

- Lecture 1 (basics of mathematical foundations) review of set theory, convexity, linear algebra, basic functional analysis, matrix norms, basic inequalities. *[Chp 1—2 of Ref 5, Chp 1 and 4.1 of Ref 2]*
- Lecture 2 (basics of measure theory 1) review of Riemann integration, limitations, statistical motivation of measure theory *[Chp 11 of Ref 5]*
- Lecture 3 (basics of measure theory 2) Lebesgue measure, abstract integration, dominated convergence theorem, monotone convergence theorem *[Chp 11 of Ref 5]*
- Lecture 4 (basics of probability theory 1) random variables, expectation, distributions, independence *[Chp 1.1—1.3 of Ref 2]*
- Lecture 5 (basics of probability theory 2) independence, conditional independence *[Chp 1.4 of Ref 2]*

Part 2: Fundamentals of Statistics

- Lecture 6: (general notions in statistics) Statistic, statistical decision theory, sufficiency *[Chp 1.1—1.3 of Ref 1]*
- Lecture 7: (bias and variance) parameter estimation, bias and variance tradeoff, prediction *[Chp 1.4—1.5 of Ref 1]*
- Lecture 8: (basic estimation theory) method-of-moments, maximum likelihood, (weighted) least squares, robust estimation *[Chp 2.1—2.2 of Ref 1]*
- Lecture 9: (Consistency) convergence mode, (uniform) law of large numbers, estimation consistency, plug-in estimator *[Chp 1.5.1, 1.5.2, 1.5.4 of Ref 2]*
- Lecture 10: (Asymptotic normality) convergence in distribution, central limit theorem, asymptotic normality *[Chp 1.5.5 of Ref 2]*

- Lecture 11: (Transformation of statistics) continuous mapping theorem, delta method, Edgeworth expansion *[Chp 1.5.3, 1.5.6 of Ref 2]*

Part 3: Classical estimation theory

- Lecture 12: (UMVUE) Unbiased estimation, sufficient and complete statistics, UMVUE *[Chp 3.1.1, 3.1.2 of Ref 2]*
- Lecture 13: (Information inequality) Fisher information, Cramer-Rao lower bound, examples in linear models *[Chp 3.1.3, 3.3.1, 3.3.2 of Ref 2]*
- Lecture 14: (Unbiased estimation for non-i.i.d. data) U-statistics, unbiased estimators in survey problems *[Chp 3.2, 3.4 of Ref 2]*

Part 4: Estimation in high dimensions

- Lecture 15: (basic concentration inequalities) Sub-gaussian and sub-exponential random variables, Hoeffding's inequality, Bernstein's inequality *[Chp 2 of Ref 3]*
- Lecture 16: (vector concentration in high dimensions) vector norm concentration, examples of high-dimensional distribution *[Chp 3.1—3.4 of Ref 3]*
- Lecture 17: (nonasymptotic bounds for random matrices) covering number, random sub-gaussian matrices, *[Chp 4.1—4.4 of Ref 3]*
- Lecture 18: (statistical applications) covariance matrix estimation, community detection *[Chp 4.5, 4.7, 5.5, 5.6 of Ref 3]*
- Lecture 19: (classical Stein's phenomenon) Motivation and heuristics, Stein's identity, consequences in linear models *[Chp 4.3.3. of Ref 2]*
- Lecture 20: (shrinkage estimation in sparse signal recovery) Motivation from wavelets, thresholding, oracle inequality *[Chp 8 of Ref 6]*
- Lecture 21: (basis pursuit and compressed sensing) Sparse recovery with linear measurements, restricted null space property, compressed sensing *[Chp 7.1, 7.2 of Ref 4]*
- Lecture 22: (LASSO and regularized method) Restricted eigenvalue, geometric intuition, variable selection consistency *[Chp 7.3—7.5 of Ref 4]*
- Lecture 23: (Spectra of random matrices) Empirical spectral distribution, weak convergence, semicircle law and MP law *[Lect 1 of Ref 7]*
- Lecture 24: (high-dimensional statistical applications) high-dimensional linear regression, spiked covariance model, sparse graph, low-rank estimation, phase transition *[Lecture note]*

[1] Mathematical Statistics: Basic Ideas and Selected Topics, Volume I, Second Edition, by Bickel and Doksum
[2] Jun Shao, Mathematical Statistics: Exercises and Solutions, Springer 2005
[3] Roman Vershynin, High-dimensional Probability: An Introduction with Applications in Data Science, Cambridge 2018

[4] Martin Wainwright, High-dimensional Statistics: A Non-Asymptotic Viewpoint, Cambridge 2019
[5] Principles of Mathematical Analysis, Walter Rudin, 3rd edition
[6] Gaussian estimation: Sequence and wavelet models, Iain Johnstone, 2019
[7] Lecture notes on random matrix theory, Charles Bordenave, 2019