Discussion 5

2.1.3

Suppose that i.i.d. $X_1, \cdots, X_n$ are i.i.d random variables following (1)$\beta(\alpha_1, \alpha_2)$ (2)$\Gamma(p, \lambda)$ (3) Weibull distribution with density $\theta a x^{a-1} exp(-\theta x^a)$ (4)Pareto distribution with density $\theta a^\theta / x^{\theta+1} (\theta > 2)$. Find the method of moments estimates of parameters based on the first two moments. *Hint:* Summary of Γ distribution and its derivatives:

- $X \sim \beta(\alpha_1, \alpha_2)$, $E(X^k) = \frac{\Gamma(\alpha_1+\alpha_2)\Gamma(\alpha_1+k)}{\Gamma(\alpha_1)\Gamma(\alpha_1+\alpha_2+k)}$.
- $X \sim \Gamma(p, \lambda)$ with density $\lambda^p x^{p-1} exp(-\lambda x)/\Gamma(p)$, $E(X^k) = \frac{\Gamma(p+k)}{\Gamma(p)\lambda^k}$.
- $X \sim Weibull(a, \theta)$, $EX^k = \Gamma(1 + k/a)\theta^{-k/a}$.
- $X \sim Pareto(a, \theta)$, $EX^k = \theta a^k / (\theta - k)$ for $k < \theta$.

2.1.5

Let $X_1, \cdots, X_n$ be the indicators of n Bernoulli trails with probability of success θ. Define $\psi : \mathbb{R}^n \times (0, 1) \to \mathbb{R}$ by

$$\psi(X_1, \cdots, X_n, \theta) = \frac{S}{\theta} - \frac{n - S}{1 - \theta}$$

where $S = \sum X_i$. Find V as defined by (2.1.3)

$$V(\theta_0, \theta) = E_{\theta_0} \psi(X, \theta)$$

and show that θ_0 is the unique solution of $V(\theta, \theta_0) = 0$. Find estimating equation estimate of θ_0.

2.2.16(b)

1. Let $\mathcal{P} = \{P_\theta : \theta \in \Theta\}, \Theta \subset \mathbb{R}^p, p \geq 1$, be a family of models for $X \in \mathcal{X} \subset \mathbb{R}^d$. Let q be a map from Θ onto $\Omega, \Omega \subset \mathbb{R}^k, 1 \leq k \leq p$. Show that if $\hat{\theta}$ is an MLE of θ, then $q(\hat{\theta})$ is an MLE of $\omega = q(\theta)$.
2. Suppose $X_1, \cdots, X_n$ are i.i.d. sampled from normal distribution $N(\mu, \sigma^2)$. Find MLE for parameters.
3. If $X_1, \cdots, X_n$ are i.i.d. sampled from log-normal distributions $LN(\mu, \sigma^2)$ with density $exp\{-(\log x - \mu)^2/\sigma^2\}/(\sqrt{2\pi}\sigma x)$. Find the MLE for the mean, $exp\{\mu + \sigma^2/2\}$.