Discussion 6

2.2.21

(kiefer-Wolfowitz) Suppose $(X_1, \cdots, X_n)$ is a sample from population with density

$$f(x, \theta) = \frac{9}{10\sigma}\varphi\left(\frac{x-\mu}{\sigma}\right) + \frac{1}{10}\varphi(x-\mu)$$

where φ is the standard normal density and $\theta = (\mu, \sigma^2) \in \Theta = \{(\mu, \sigma^2) : -\infty < \mu < \infty, 0 < \sigma^2 < \infty\}$. Show that maximum likelihood estimates do not exist, but that $\sup_\sigma p(x, \hat{\mu}, \sigma^2) = \sup_{\mu,\sigma^2} p(x, \mu, \sigma^2)$ if, and only if, $\hat{\mu}$ equals one of the numbers $x_1, \cdots, x_n$. Assume that $x_i \neq x_j$ for $i \neq j$ and that $n \geq 2$.

2.2.39

Let X_i denote the number of hits at a certain Web site on day i, $i = 1, \cdots, n$. Assume that $S = \sum_{i=1}^n X_i$ had a Poisson, $\mathcal{P}(n\lambda)$, distribution. On day $n+1$ the Web Master decides to keep track of two types of hits(money making and not money making). Let V_j and W_j denote the number of hits of type 1 and 2 on day j, $j = n+1, \cdots, n+m$. Assume that $S_1 = \sum_{j=n+1}^{n+m} V_j$ and $S_2 = \sum_{j=n+1}^{n+m} W_j$ have $\mathcal{P}(m\lambda_1)$ and $\mathcal{P}(m\lambda_2)$ distributions, where $\lambda_1 + \lambda_2 = \lambda$. Also assume that S, S_1 and S_2 are independent. Find the MLEs of λ_1 and λ_2 based on S, S_1 and S_2.

2.2.40

Let $X_1, \cdots, X_n$ be a sample from the generalized Laplace distribution with density

$$f(x, \theta_1, \theta_2) = \begin{cases} \dfrac{1}{\theta_1 + \theta_2}\exp\{-x/\theta_1\}, & x > 0 \\ \dfrac{1}{\theta_1 + \theta_2}\exp\{x/\theta_2\}, & x < 0 \end{cases}$$

where $\theta_j > 0$, $j = 1, 2$.

1. Show that $T_1 = \sum X_i 1(X_i > 0)$ and $T_2 = \sum -X_i 1(X_i < 0)$ are sufficient statistics.
2. Find the maximum likelihood estimates of θ_1 and θ_2 in terms of T_1 and T_2. Carefully check the "$T_1 = 0$ or $T_2 = 0$" case.