Discussion 7

We will discussion of several extensions of Example 2.4.6. Remember that we have (Z_i, Y_i) be i.i.d. following $\mathcal{N}(\mu_1, \mu_2, \sigma_1^2, \sigma_2^2, \rho)$ bivariate normal distribution, $i = 1, 2, \cdots, n$. We observe both Z_i and Y_i for $1 \leq i \leq n_1$. We miss Y_i's when $n_1 + 1 \leq i \leq n_2$, and miss Z_i's when $n_2 + 1 \leq i \leq n$.

2.4.1

EM for bivariate data. $(Z_i, Y_i) \sim N(\mu_1, \mu_2, \sigma_1^2, \sigma_2^2, \rho)$

- In the bivariate normal Example 2.4.6, Complete the E-step by finding $E(Z_i|Y_i)$, $E(Z_i^2|Y_i)$ and $E(Z_iY_i|Y_i)$.
- In Example 2.4.6, verify the M-step by showing that

$$E_\theta(\mathbf{T}) = (\mu_1, \mu_2, \sigma_1^2 + \mu_1^2, \sigma_2^2 + \mu_2^2, \rho\sigma_1\sigma_2 + \mu_1\mu_2)$$

2.4.17

Limitations of the missing value model of Example 2.4.6. In Example 2.4.6, suppose Y_i is missing iff $Y_i \geq 2$. If $\mu_2 = 1.5$, $\sigma_1 = \sigma_2 = 1$ and $\rho = 0.5$, find the probability that $E(Y_i|Z_i)$ underpredicts Y_i.

3.2.9

Suppose we have a sample $X_1, \cdots, X_n$ of differences in the effect of generic and name-brand effects for a certain drug, where $E(X) = \theta$. A regulatory agency specifies a number $\epsilon > 0$ such that $\theta \in (-\epsilon, \epsilon)$, then the generic and name-brand drugs are, by definition, bioequivalent. On the basis of $\mathbf{X} = (X_1, \cdots, X_n)$ we want to decide whether or not $\theta \in (-\epsilon, \epsilon)$. Assume that given θ, $X_1, \cdots, X_n$ are i.i.d. $N(\theta, \sigma_0^2)$, where σ_0^2 is known, and that θ is random with a $N(\eta_0, \tau_0^2)$ distribution. There are two possible actions:

$$a = 0 \Leftrightarrow \text{Bioequivalent}$$
$$a = 1 \Leftrightarrow \text{Not Bioequivalent}$$

with losses $l(\theta, 0)$ and $l(\theta, 1)$. Set

$$\lambda(\theta) = l(\theta, 0) - l(\theta, 1) = r - \exp\left\{-\frac{1}{2c^2}\theta^2\right\}, \quad c^2 > 0.$$

where $0 < r < 1$ and $\log r = -\frac{1}{2c^2}\epsilon^2$.

1. Show that the Bayes rule is equivalent to

"Accept bioequivalence if $E(\lambda(\theta)|X = x) < 0$"

and show that it is equivalent to

$$\text{“Accept bioequivalence if } [E(\theta|x)]^2 < \left(\tau_0^2(n) + c^2\right)\left\{\log\left(\frac{c^2}{\tau_0^2(n) + c^2}\right) + \frac{\epsilon^2}{c^2}\right\}\text{”}$$

where

$$E(\theta|x) = w\eta_0 + (1-w)\bar{x}, \quad w = \tau_0^2(n)/\tau_0^2, \quad \tau_0^2(n) = \left(\frac{1}{\tau_0^2} + \frac{n}{\sigma_0^2}\right)^{-1}.$$

2. It's proposed that the preceding prior is "uninformative" if it has $\eta_0 = 0$ and τ_0^2 large (“$\tau_0^2 \to \infty$”). Discuss the preceding decision rule for this “prior”.

3. Discuss the behavior of the preceding decision rule for large n(“$n \to \infty$”). Consider the general case (a) and specific case (b).