Discussion 10

4.1.2

Assume $X = (X_1, \cdots, X_n)$ is an Exp(λ) sample. Consider the hypothesis $H : 1/\lambda = \mu < \mu_0$.

1. Show that the test with critical region

$$\left[\bar{X} \geq \frac{\mu_0}{2n}\chi^2_{2n,1-\alpha}\right]$$

   where $\chi^2_{2n,1-\alpha}$ is the (1-α)th quantile of the χ^2_{2n} distribution, is a size α test.

2. Give an expression of the power in terms of the χ^2_{2n} distribution.

3. Use the central limit theorem to show that $\Phi[\mu_0 z(\alpha)/\mu) + \sqrt{n}(\mu - \mu_0)/\mu]$ is an approximation to the power of the test in part(a).

3.5.5

Show that the a trimmed mean

$$\bar{X}_\alpha = \frac{X_{([n\alpha]+1)} + \cdots + X_{(n-[n\alpha])}}{n - 2[n\alpha]}$$

is an empirical plug-in estimate of

$$\mu_\alpha = \frac{1}{1-2\alpha}\int_{x_{1-\alpha}}^{x_\alpha} x dF(x).$$

Here $\int x dF(x)$ denotes $\int xp(x)dx$ in the continuous case and $\sum xp(x)$ in the discrete case.

4.1.11

In Example 4.1.5, let $\psi(\mu)$ be a function from (0, 1) to (0, ∞), and let $\alpha > 0$. Define the statistics

$$S_{\psi,\alpha} = \sup_x \psi(F_0(x))|\hat{F}(x) - F_0(x)|^\alpha$$

$$T_{\psi,\alpha} = \sup_x \psi(\hat{F}(x))|\hat{F}(x) - F_0(x)|^\alpha$$

$$U_{\psi,\alpha} = \int \psi(F_0(x))|\hat{F}(x) - F_0(x)|^\alpha dF_0(x)$$

$$V_{\psi,\alpha} = \int \psi(\hat{F}(x))|\hat{F}(x) - F_0(x)|^\alpha d\hat{F}(x)$$

1. For each of these statistics show that the distribution under H does not depend on F_0.

2. When $\psi(u) = 1$ and $\alpha = 2$, $V_{\psi,\alpha}$ is called the Cramer-von Mises statistic. Express the Cramer-von Mises statistic as a sum.

3. Are any of the four statistics in (a) invariant under location and scale?