STAT 610 DISCUSSION 11

1. **Summary**

- **Likelihood ratio test**
  - *Likelihood ratio statistic* is defined by $L(x, \theta_0, \theta_1) = \dfrac{p(x, \theta_1)}{p(x, \theta_0)}$.
  - *Likelihood ratio test* is a test that for some $0 \leq k \leq \infty$, we can write the test function φ_k as

$$\varphi_k(x) = \begin{cases} 1 & \text{if } L(x, \theta_0, \theta_1) > k \\ 0 & \text{if } L(x, \theta_0, \theta_1) < k \end{cases}$$

  - **Theorem 4.2.1** (Neyman-Pearson Lemma)
    (a) If $\alpha > 0$ and φ_k is a size α likelihood ratio test, then φ_k is MP in the class of level α tests.
    (b) For each $0 \leq \alpha \leq 1$ there exists an MP size α likelihood ratio test provided that randomization is permitted, $0 < \varphi < 1$, for some x.
    (c) If φ is an MP level α test, then it must be a level α likelihood ratio test; that is , there exists k such that

$$P_\theta[\varphi(X) \neq \varphi_k(X), L(X, \theta_0, \theta_1) \neq k] = 0$$

    for $\theta = \theta_0$ and $\theta = \theta_1$.

- **Monotone Likelihood Ratio Model**
  - The family of models $\{P_\theta : \theta \in \Theta\}$ with $\Theta \subset R$ is said to be a *monotone likelihood ratio* (MLR) family in T if for $\theta_1 < \theta_2$ the distributions P_{θ_1} and P_{θ_2} are distinct and there exists a statistic $T(x)$ such that the ratio $p(x, \theta_2)/p(x, \theta_1)$ is an increasing function of $T(x)$.
  - Define $\delta_t(x) = \begin{cases} 1 & \text{if } T(x) > t \\ 0 & \text{if } T(x) < t \end{cases}$
  - **Theorem 4.3.1** Suppose $\{P_\theta : \theta \in \Theta\}$, $\Theta \subset R$, is an MLR family in $T(x)$.
    (a) For each $t \in (0, \infty)$, the power function $\beta(\theta) = E_\theta \delta_t(X)$ is increasing in θ.
    (b) If $E_\theta \delta_t(X) = \alpha > 0$, then δ_t is UMP level α for testing $H : \theta \leq \theta_0$ versus $K : \theta > \theta_0$.

2. **Examples**

- Example 1 (4.2.5): A newly discovered skull has cranial measurements (X, Y) known to be distributed either (as in population 0) according to N(0,0,1,1,0.6) or (as in population 1) according to N(1,1,1,1,0.6) where all parameters are known. Find a statistic $T(X, Y)$ and a critical value c such that if we use the classification rule, (X, Y) belongs to population 1 if $T \geq c$, and to population 0 if $T < c$, then the maximum of the two *probabilities of misclassification* $P_0[T \geq c], P_1[T < c]$ is as small as possible.
- Example 2 (4.2.7): Prove (Corollary 4.2.1.) that if φ is an MP level α test, then $E_{\theta_1}\varphi(X) \geq \alpha$ with equality iff $p(\cdot, \theta_0) = p(\cdot, \theta_1)$.