Discussion 12

Confidence bounds, intervals and regions

4.5.1

Let $X_1, \cdots, X_{n_1}$ and $Y_1, \cdots, Y_{n_2}$ be independent exponential $\mathcal{E}(\theta)$ and $\mathcal{E}(\lambda)$ samples, respectively, and let $\Delta = \theta/\lambda$.

1. If $f(\alpha)$ denotes the αth quantile of $F_{2n_1,2n_2}$ distribution, show that $[\bar{Y} f(\frac{1}{2}\alpha)/\bar{X}, \bar{Y} f(1 - \frac{1}{2}\alpha)/\bar{X}]$ is a confidence interval for Δ with confidence coefficient $1 - \alpha$.
2. Show that the test with acceptance region $[f(\frac{1}{2}\alpha) \leq \bar{X}/\bar{Y} \leq f(1 - \frac{1}{2}\alpha)]$ has size α for testing $H : \Delta = 1$ versus $K : \Delta \neq 1$.

4.6.1

Suppose $X_1, \cdots, X_n$ is a sample from a $\Gamma(p, \frac{1}{\theta})$ distribution, where p is known and θ is unknown. Exhibit the UMA level $(1 - \alpha)$ UCB for θ.

Frequentist and Bayesian formulations

4.7.1

1. Show that if $\boldsymbol{\theta}$ has a beta, $\beta(r, s)$, distribution with r and s positive integers, then $\lambda = \frac{s\theta}{r(1-\theta)}$ has the F distribution $F_{2r,2s}$.
2. Suppose that given $\boldsymbol{\theta} = \theta$, X has a binomial, $\mathcal{B}(\backslash, \theta)$, distribution and that θ has beta, $\beta(r, s)$ distribution with r and s integers. Show how the quantiles of the F distribution can be used to find upper and lower credible bounds for λ and for θ.

Prediction intervals

4.8.2

Let $X_1, \cdots, X_{n+1}$ be i.i.d. as $X \sim F$, where $X_1, \cdots, X_n$ are observable and X_{n+1} is to be predicted. A level $(1 - \alpha)$ lower(upper) prediction bound on $Y = X_{n+1}$ is defined to be a function $\underline{Y}(\overline{Y})$ of $X_1, \cdots, X_n$ such that $P(\underline{Y} \leq Y) \geq 1 - \alpha$ $(P(Y \leq \overline{Y}) \geq 1 - \alpha)$.

1. If F is $N(\mu, \sigma_0^2)$ with σ_0^2 known, give level $(1 - \alpha)$ lower and upper prediction bound for X_{n+1}.
2. If F is $N(\mu, \sigma^2)$ with σ^2 unknown, give level $(1 - \alpha)$ lower and upper prediction bound for X_{n+1}.
3. If F is continuous with a positive density f on (a, b), $-\infty \leq a < b \leq \infty$, give level $(1 - \alpha)$ distribution free lower and upper prediction bounds for X_{n+1}.