Discussion 8

5.3.8

Let $X_1, \cdots, X_{n_1}$ be i.i.d. F and $Y_1, \cdots, Y_{n_2}$ be i.i.d. G, and suppose the X's and Y's are independent.

1. Show that if F and G are $N(\mu_1, \sigma_1^2)$ and $N(\mu_2, \sigma_2^2)$, respectively, then the LR test of $H : \sigma_1^2 = \sigma_2^2$ versus $K : \sigma_1^2 \neq \sigma_2^2$ is based on the statistic s_1^2/s_2^2, where $s_1^2 = (n_1 - 1)^{-1}\sum_{i=1}^{n_1}(X_i - \bar{X})^2, s_2^2 = (n_2 - 1)^{-1}\sum_{i=1}^{n_2}(Y_i - \bar{Y})^2$.

2. Show that when F and G are normal as in part (a), then $(s_1^2/\sigma_1^2)/(s_2^2/\sigma_2^2)$ has an $F_{k,m}$ distribution with $k = n_1 - 1$ and $m = n_2 - 1$.

3. Now suppose that F and G are not necessarily normal but that

   $$G \in \mathcal{G} = \left\{ F\left(\frac{\cdot - a}{b}\right) : a \in R, b > 0 \right\}$$

   and that $0 < \text{Var}(X_1^2) < \infty$. Show that if $m = \lambda k$ for some $\lambda > 0$ and

   $$c_{k,m} = 1 + \sqrt{\frac{\kappa(k+m)}{km}} z_{1-\alpha}, \kappa = \text{Var}[(X_1 - \mu_1)/\sigma_1]^2, \mu_1 = E(X_1), \sigma_1^2 = \text{Var}(X_1).$$

   Then, under $H : \text{Var}(X_1) = \text{Var}(Y_1)$, $P(s_1^2/s_2^2 \leq c_{k,m}) \to 1 - \alpha$ as $k \to \infty$.

4. Let $\hat{c}_{k,m}$ be $c_{k,m}$ with κ replaced by its method of moments estimate. Show that under assumptions of part(c), if $0 < EX_1^8 < \infty$, $P_H(s_1^2/s_2^2 \leq \hat{c}_{k,m}) \to 1 - \alpha$ as $k \to \infty$.

5.3.17

Suppose $X_1, \cdots, X_n$ are independent, each with Hardy-Weinberg frequency function f given by

x	0	1	2
$f(x)$	θ^2	$2\theta(1-\theta)$	$(1-\theta)^2$

where $0 < \theta < 1$.

1. Find an approximation to $P[\bar{X} \leq t]$ in terms of θ and t.

2. Find an approximation to $P[\sqrt{\bar{X}} \leq t]$ in terms of θ and t.

3. What is approximate distribution of $\sqrt{n}(\bar{X} - \mu) + \bar{X}^2$, where $\mu = E(X_1)$?